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A Problem to Solve

CHILDHOOD EDUCATION

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A Problem to Solve

From a photograph by Lena Towsley

When to Begin the Teaching of Arithmetic

B. R. BUCKINGHAM

Ginn and Company, Boston, Massachusetts

THE question raised by the title of this article is a part of the larger problem of grade placement. It is my purpose to point out why the question arises in this particular form, what kind of arithmetic we may properly consider in seeking an answer to the question, and where we ought to look for the answer.

WHY THE QUESTION ARISES

It is generally agreed that arithmetic is hard. Many writers have pointed out that more pupils fail in it than in any other elementary-school subject. A critical examination of the evidence is, for our present purpose, neither necessary nor appropriate. We may take it for granted that arithmetic, at least in some of its aspects and as generally administered, is so exacting as to raise seriously the question of its appropriateness as a subject in the lower grades.

The result is that there has been a movement to postpone the beginning of arithmetic from the first grade to the second or third. In one sense this is a strange phenomenon. It is not only the sole instance of a tendency to delay the teaching of a subject but it actually runs counter to a much stronger opposite trend. In other words, so far as the shifting of subject matter is concerned, most subjects are gradually moving into lower rather than higher grades. Geography has worked its way down into the third grade. History is studied in the fourth grade. And science is now offered in the primary grades. Arithmetic alone of all the content subjects tends to move in the other direction. While other subjects are being adapted to immature students and while, riding the wave of better teaching of reading, these other subjects are enriching younger lives than ever before, this subject remains hard and inflexible, demanding for itself concessions which are accorded nowhere else.

The postponement of which I have spoken has often been the subject of study either observational or experimental. In particular the deferring of arithmetic to the third grade has attracted attention. The procedure in this respect has been uniform and the reports, so far as they have been published, have been in substantial agreement.

Where experimental conditions have been set up, one group of children has been taught arithmetic in the first and second grades whereas another group, presumably or demonstrably of the same ability has received its first instruction upon entering grade three. At some subsequent period, say at the end of grade three, both groups have been given the same arithmetic test and each has been found to be of approximately the

same attainment as measured by the test. The inference has been that teaching arithmetic in grades one and two is a waste of time. I shall defer until later a consideration of the validity of this inference.

Meanwhile, however, my immediate purpose is to show why we have before us this question of the grade in which arithmetic should be begun. It is first because the subject has been found to be hard and secondly because, strangely enough, there has been a movement not, as in other cases, to adapt the material to the child but rather to adapt the child to the material. This has been accomplished by keeping the subject matter fixed and demanding a certain mental maturity before contact with it is permitted.

What Kind of Arithmetic Do We Mean?

The term arithmetic even when we try to give it a scientific definition covers scores of major topics and hundreds, perhaps thousands, of skills. It is perfectly possible for a child to be good in one aspect of arithmetic and not in another. This condition is abundantly apparent in school work. Children may be successful with abstract work and fail in the solution of verbal problems. Standardized tests have also brought out unsuspected inequalities in a given student's performance within abilities usually thought of as unitary.

Among the many divisions which we may make of arithmetic we may recognize concrete and abstract areas. Counting objects, reproducing, matching, and identifying numbers by means of objects, solving verbal problems by actual measurement with a ruler or with pint and quart cups, reaching conclusions on the basis of one-to-one correspondence, dramatizing a quantitative situation with a consequent decision as to its meaning, putting together or adding groups of objects, taking them apart or subtracting them, playing games which require the use of number ideas, manipulating objects or pictures or number patterns for a quantitative purpose—all these are as truly "arithmetic" as abstract number facts and processes.

When therefore we raise the question of the grade in which to begin arithmetic it is proper to ask, what kind of arithmetic do we mean? It is true that the ordinary course of study for grades one and two is heavily weighted with abstract counting and with number facts and processes. But does this need to be true?

I think it is safe to say that in the minds of most students of arithmetic today the answer is that it need not be true. Moreover I think most of these students are agreed that it ought not to be true. The usual procedure in the grade where arithmetic begins is for the pupils first to count objects—an admirably concrete activity, but generally one which offers no challenge because children are already abundantly able to do it. Almost at once after this counting of objects the pupils are required to learn the number facts in their abstract form. The rest is drill.

This is the program which proves so unsatisfactory. This is the program which causes so many failures in arithmetic. This is the program which causes some school people to postpone arithmetic to a higher grade, demanding adjustment of the child rather than of the subject. They realize that the correct handling of abstract numbers is a relatively mature process. They therefore delay it until the child has become more mature. They are in reality turning over to life outside of school the learning which should take place in school but which generally does not—namely, the learning of the rich field of concrete arithmetic which lies between counting and the manipulation of abstract number.

In his admirable book on the teaching of history Henry Johnson points out that history can be taught in the primary grades—that is, real history according to the most rigorous definition of the term—history which is true and which does not have to be re-learned when the child reaches a period of greater maturity, history which fortifies and makes meaningful the learning of the concepts in more mature courses. This primary-grade history, Professor Johnson points out, is distinguished from history for later grades by being dominantly *concrete*.

The idea is equally applicable to arithmetic. Here we have violated every canon to which we subscribe in other subjects by hurrying the child too quickly into abstractions for which he is not ready. The inevitable result is failure with all its concomitants of discouragement and dissatisfaction. Children actually learn—for they learn attitudes along with their arithmetic—to despise the subject and to avoid mathematics in general. In my judgment there is no single reason for this aversion to arithmetic more potent than the fact that we have made it abstract at a time when abstraction of this sort means little or nothing to the child.

If therefore one says that arithmetic is inappropriate for grade one, we shall so far agree with him as to say that arithmetic as it is now being taught is inappropriate for that grade. We shall point out, however, that there are vast ranges of concrete arithmetic which are quite different in their appeal to young children. They are still indubitably arithmetic. They do not necessitate any false moves now or later. They require no subsequent changes in the form or substance of the subject. Moreover the teaching of concrete arithmetic will strongly fortify abstract arithmetic. It will do in a planned, orderly, and relatively complete manner what those who postpone arithmetic to the third grade turn over to life outside of school to be done piecemeal, haphazardly, and incompletely.

Where to Look for an Answer

So far we have failed to find—in fact we have not sought—an answer to the question which constitutes the title of this article. We have said that the question arises because arithmetic is hard and that doubt has been cast upon the old established course by experiments in the postponement of arithmetic. We have likewise indicated that arithmetic is varied and that among its variations we may distinguish concrete from abstract. We have pointed out too that concrete arithmetic is true arithmetic, that it may be offered to young children, and that if so offered it will buttress abstract arithmetic.

Let us now look more definitely for the answer to our question. To my mind two standards and only two apply to the grade placement of curriculum material—first, is the child ready for it; secondly, is it socially useful in his present and later life? At this point it would be easy to discuss other criteria—interest, for example, and opportunity for desirable growth. A course, however, for which the children are intellectually and emotionally ready and which has material that meets their needs will not only be interesting to them but will lead them to further interests and thus provide for growth. I have no objection whatever to the setting up of other criteria nor do I believe that the argument which I propose to base upon these two, namely, readiness and social utility, will be in any way invalidated through the use of additional standards.

No more important dictum is to be found in pedagogy than this: Meet the child where he is. Don't meet him where you think he is. Know where he is. Don't meet him where he ought to be or where his mother says he is. It is important that there should be no mistake about the interests and abilities of the pupil whom we are taking in hand.

This is the question of readiness. Fortunately we have a respectable body of information as it applies to children who have not yet been taught number at school, especially to children upon entering grade one. This information is more coherent, more consistent, than most of the information we possess in education. The child does not come to school ignorant of number. His mind is not a blank with reference to quantity. He has already learned a surprising amount of arithmetic. He is already able to act with considerable assurance and versatility under the guidance of quantitative ideas.

On entering grade one at the age of six the child can count to 20 either by rote or with objects. There is at least a fifty-fifty chance that he can count by 10's as far as 40. He knows 5 and 6 in the sense that he can make up groups of objects of these specified numbers or that he can identify groups having that many objects. He is nearly as capable

in his knowledge of the other numbers to 10. If you give him concrete problems involving addition combinations whose sums do not exceed 10 he will give you the right response often enough to show that problems of this sort are beginning to have meaning for him. When the two components of a number are shown objectively and then concealed he can give the invisible sum with still greater success.

In this connection I am reminded of a paragraph which William L. Connor, Chief of the Bureau of Educational Research at Cleveland, wrote after having tested the number knowledge of kindergarten children:

> The amount of number used by little children, as revealed by this test, has proved to be a revelation to all of those connected with the teaching and supervision of little children in the Cleveland schools. It seems not at all improbable that the fear that school officers seem to have had of crowding number concepts on little children too soon is wholly without foundation in fact. When one considers the ordinary teaching materials in arithmetic in the light of these findings, fragmentary as they are, it would almost seem as if the course-of-study makers, textbook writers, and teachers of the last generation had conspired to retard by every means possible the natural growth of number concepts which takes place in little children from four to seven years of age.[1]

This is a strong statement; but the evidence sustains it. The testimony of the school people at Cincinnati, the testimony of Miss Ada R. Polkinghorne in her study of the knowledge of fractions on the part of young children, the testimony of Professor Clifford Woody who likewise tested children in the primary grades before they had received any instruction in school—this testimony plus an accumulation of data in Germany, England, France, and Belgium proves beyond a reasonable doubt that children have already learned a great deal of number before we begin to teach it to them and specifically that children six years of age upon entering grade one are ready to learn number provided the right kind of number is offered to them.

After one has considered the readiness of six-year-old children for number experience, it is proper to seek an answer to our question by inquiring into the social utility of arithmetic for children of this age. I am going to spare the reader a disquisition on the general question of the social value of arithmetic. I have expatiated on that topic elsewhere. The simple inquiry with which I feel we may now be legitimately concerned is this: Does the child—the six-year-old and the seven-year-old child, to be specific—use number for his own purposes? Anyone who has casually observed the behavior of children knows that they use a great deal of concrete arithmetic. Miss Marion J. Wesley, in another article of this issue, has some pertinent statements to make on this matter. Miss Nila B. Smith, reporting the results of first-hand knowledge of the behavior of five hundred first-grade children at Detroit found that in their out-of-school lives they made an amazing use of arithmetic.[2] They counted, added, subtracted, used fractions and numerals; they measured and compared; they multiplied and divided. The more frequent activities in which they used these number ideas and processes were classified by Miss Smith as follows: transactions carried on in stores, games involving counting, reading Roman numerals on the clock, reading Arabic numerals in finding pages in books, dividing food with playmates and pets (fractions), playing store, depositing money in and drawing money from toy banks, measuring distance, using calendars, running errands, setting the table, buying and selling tickets, acting as newsboy, measuring in sewing.

Anyone who knows children will recognize the authenticity of this list of activities and will grant the probable need for number in pursuing them.

In our search for an answer to the question when to begin arithmetic, we are now confronted by two pertinent facts: first, that the child upon entering grade one is ready for arithmetic; and secondly, that he is already using it to serve his own purposes and will continue to do so independently of any action taken by the school. Arithmetic has social utility for him. It enables him more fully to satisfy his normal purposes.

[1] *Twenty-ninth Yearbook of the National Society for the Study of Education*, p. 512.

[2] Smith, Nila B. "An Investigation of the Uses of Arithmetic in the Out-of-School Life of First-Grade Children." *Elementary School Journal*, 24: 621–626, April, 1924.

Accordingly the answer to our question comes almost automatically. If the time to begin is when the child is ready and has use for the subject, then *we should begin the teaching of arithmetic as soon as he comes to school.*

Although the argument as here presented is intended to apply only to the *graded* school, that is to the school which begins with grade one, the conclusion as stated is almost equally applicable to the kindergarten. Those who are unacquainted with modern kindergarten procedure may be surprised to learn that there is already a great deal of arithmetic in it. Moreover first-grade pupils benefit by it. In the investigation as to the number knowledge of first-grade children which Doctor MacLatchy and I conducted, children who had received the benefit of kindergarten training were superior to those who had not received such training in every test which we employed. This can only mean that kindergarten experience calls for the use of number.

Sometimes the question to which we are addressing ourselves is posed in this way: When should the *formal* teaching of arithmetic begin? It is possible in reply to this question to answer "Never." Formalism is as repugnant to the teaching of arithmetic as it is to the teaching of any other subject. But if by formal we mean systematic, regular, provided for in the schedule, then the answer still remains as before. Teach arithmetic from the time the child enters school. Incidental teaching has its advocates. Few of them, however, apply it to arithmetic. Here is a subject which more than any other in the elementary school is organized and sequential. Let the teacher accomplish all she can through incidental learning. But let it never be supposed that either in the first grade or elsewhere a satisfactory course in arithmetic can be provided without being planned and systematically pursued.

What then of the experiments which have been supposed to prove that teaching arithmetic in the first and second grades was a waste of time? These experiments simply have not proved what people, including the experimenters, have concluded that they proved. Let us imagine the situation. In a given school we divide the children upon entering grade one into two groups of equal ability. We teach arithmetic to one group throughout the first and second grades. We teach no arithmetic to the other group during their attendance upon these grades. Both groups are taught arithmetic during the third grade and at the end of that grade you can't tell one group from the other.

These facts do not make sense. I do not mean that the tests at the end of the third grade did not yield the measures which were reported. What I do mean is that without intention the dice were loaded. When the two groups, the taught and the untaught, entered the third grade the taught group was unquestionably superior to the untaught group in arithmetic. Why did it not remain so? Possibly the kind of arithmetic offered to these children was inappropriate —such, for example, as could not be expected to remain with them as a permanent possession. Unduly abstract material would be likely to be ineffective in precisely this way. But even this merely means that a more appropriate type of arithmetic should have been offered—a type in other words which would have been retained. Waiving, however, the question of the appropriateness of the particular kind of arithmetic which the taught group had received, it is still clear that for whatever work either group would be likely to do in grade three the taught group was superior. Here was something to build upon. And if it were built upon, it is clear that the taught group at the end of grade three would remain superior to the untaught group. In fact, if the same energy and resourcefulness were devoted to the taught group as to the untaught group during the third grade it is probable that the two groups would be wider apart at the end of the year than they were at the begining of it.

No, the experimental evidence does not mean what it has been held to mean. It does not mean that instruction in arithmetic may be neglected for two years. It means nothing so clearly as that the children who were taught arithmetic in grades one and two and who were allowed to fall back to the level of those who began arithmetic in grade three have been defrauded.

Number Abilities of First-Grade Children

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FIRST-GRADE teachers have always known that some children are familiar with a few number facts when they enter first grade. The extent of this early familiarity and its prevalence among six-year-old children were not recognized until the answers given by twenty-three hundred six-year-olds to a simple number test were summarized several years ago.[1] These six-year-olds entered the first grades of eleven cities of Ohio in September, 1928.[2]

Because the children were unable to write and were unaccustomed to testing procedures, each was questioned individually by his teacher. Since these teachers, like all teachers, were unusually busy during the first two weeks of school, each was asked to test only six of her pupils—the first six in the alphabetical list of pupils' surnames whose birth dates fell between March 1 and September 1, 1922.

This method of selection assured a random sample of children entering school for the first time. Further evidence of this randomness came from the records which showed no differences traceable to the section within a city, to the kind of community, or to the part of the state in which the children lived. For these reasons the percentage summaries of the answers of this large group may be considered typical for the mass of six-year-old children who enter school each September.

Instead of the percentage summaries of the Ohio children's answers, the percentages are expressed here as children in an imaginary six-year-old group. We shall suppose that there are thirty-five children in this group, some of whom have attended kindergarten a year, and none of whom have been in Grade 1 before. The abilities of this unselected first grade to count by rote will be something like this:

One child will not know how to count
One child will count to 5
Three children will count to 10
Nine children will count to 15
Nine children will count to 20
Two children will count to 30
Three children will count to 40
Two children will count to 50
Two children will be uncertain counters from 50 to 99
Three children will count to 100

There are variations of the ability and of the experience of the children which may affect the number knowledge of the group. If all the children have attended kindergarten, then they all will be able to count to 10 and five of them will count to 100. If none of the group has attended kindergarten, two of them will not be able to count at all, ten of them will not be able to count beyond 10, and only one will count to 100.

If our imaginary group belongs to a school system in which the pupils are assigned to particular groups by some measure of their intelligence, the group may be composed of superior, average, or slow six-year-olds. If the 35 children in this first grade have been classified as superior in intelligence, one of them may not be able to count, four will be able to count to 10, and six will be able to count to 100. One of the 35 children in the group of average ability will not be able to count, seven will be able to count to 10, and one will be able to count to 100. Four of the 35 children in the slow group will not be able to count, eleven will not be able to count beyond 10, and no child will be able to count to 100.[3]

[1] Buckingham, B. R. and MacLatchy, Josephine H. "The Number Abilities of Children When They Enter Grade One." *Twenty-ninth Yearbook of the National Society for the Study of Education*, pp. 473–524, 1930.

[2] The cities were: Cincinnati, Lakewood, Cleveland Heights, Akron, Toledo, Hamilton, Portsmouth, Lima, Kent, Marietta, and Upper Arlington.

[3] For these and similar descriptions the summary records of the Cincinnati children classified according to their percentile ratios on the Pintner-Cunningham intelligence test were used. The percentages for the children whose percentile ratios ranged from 100 to 76 were used for the superior group; those from 75 to 26 for the average group; and those from 25 to 1 for the slow group. MacLatchy, Josephine H. "A Phase of First-Grade Readiness." *Educational Research Bulletin*, 10: 377–380, October 14, 1931.

Twenty-one of the children of the typical first grade will understand the one-to-one, the enumerative relation in counting 20 objects. One will not understand it at all, and three will not be able to count 10 objects.

Four of the thirty-five children in the typical first grade will not be familiar with addition: they will not be able to recall the sums of any pairs of numbers in simple problems, nor will they be able to determine by counting the sum of two small groups of objects put before them. At the other extreme will be the two children who will likely know all the combinations with sums of 10 or less.

Suppose that children of this hypothetical first grade are asked the sums of the 10 combinations given to the Ohio children in problems put in this form, "If you have 5 pencils and you get 1 more, how many pencils will you have then?" their familiarity with addition will appear something like this:

Number of Combinations Known	*Likely the Familiar Combinations*	*Number of Children Knowing*
0	.....	4
1	5+1	4
2	That just given and 7+1	4
3	Those just given and 1+9	4
4	Those just given and 1+6	3
5	Those just given and 5+2	4
6	Those just given and 8+2	3
7	Those just given and 4+4	3
8	Those just given and 5+3	2
9	Those just given and 3+5	2
10	Those just given and 4+5	2

The word "likely" in the caption, "Likely the Familiar Combinations," over the second column of the table just given is important. The Ohio children's familiarity with addition did not seem to be systematic. At least, it was not possible to predict that because a child knew the sum of 5+1 he would know any other combination. Then, too, the Ohio children's familiarity with addition did not follow the order given in any of the lists of combinations which have been compiled to show the relative difficulty of addition combinations. Two trends seemed evident, however. The first was that their familiarity with addition was closely related to counting. Thus, 3+1, 5+1, 6+1, 7+1 and 8+1 were familiar to larger percentages of the group than were 1+7, 1+6, and 1+9; and 2+2, 5+2, and 8+2 were known by larger percentages than were 2+6, 2+4, and 2+8.[4] Then, too, there seemed to a tendency for the percentages of familiarity for the combinations having the same sum to fall rather close together. This numerical picture of the familiarity with number which is characteristic of a group of six-year-olds puts individual differences in the foreground. The evidence is no argument for the beginning of formal instruction in arithmetic, but it shows the necessity of great care and wisdom in bringing these heterogeneous familiarities to a place of common understanding.

Where did these six-year-olds learn what they knew about number? Some of them learned some of the facts in kindergarten. But some of them did not go to kindergarten, and in the detailed records of the Ohio groups there were no number facts known by children who had attended kindergarten for a year that were not known by some children who had not gone to kindergarten. The answer to our question, then, is that number facts may be learned by children without kindergarten instruction, which, even though it is incidental is more systematic than is the unpremeditated learning which takes place in a child's everyday experience.

Evidence of this early learning of number has been reported by several investigators. Binet reported in 1890 that his four-year-old daughter could perceive the number 4 but that his two-year-old daughter could not "recognize with any assurance numbers beyond 3."[5] Decroly records evidence of his little daughter's recognition of the number in a group at fourteen months.[6] He con-

[4] Since only 20 of the combinations with sums of 10 or less were used, there are many combinations which did not appear in the test, and whose familiarity for the Ohio six-year-olds is, therefore, not known. The combinations mentioned here are only those used in the "Number Test for Pupils Entering Grade One." MacLatchy, Josephine H. "Another Measure of the Difficulty of Addition Combinations." *Educational Research Bulletin*, 12: 57–61, March 8, 1933.

[5] Binet, Alfred. "La Perception des Longuers et des Nombres." *Revue Philosophique*, 30: 68–81, Juillet, 1890.

[6] Dercroly, M. Le Dr. and Degand, Mlle. Julia. "Observations Relatives à l'Évolution des Notions de Quantites, Continues et Discontinues Chez l'Enfant." *Archives de Psychologie*, 12: 81–121, May, 1912.

tinued a diary account of her advance in number familiarity until she was five years old. He summarized his findings[7] as follows:

> . . . the child uses the names of numbers at first only mechanically. Before knowing how to count, he is already capable of accounting for the constitution of groups; counting itself is only done awkwardly at the beginning, the child not being able to go beyond "two" (at about two years of age); then, a long time later, "three," whereas quantities above this number are indicated at random. Further, we observe the preparatory step in which the child accounts for the presence or the absence of things.
>
> Then we see an interesting and absolutely logical step, the idea of "two" preceding the numerical idea of "one." . . . We see also the moment when the question, "How many?" is understood, then the moment when it is used by the child first with a utilitarian end, later with speculative purpose. The appearance of "three," "four," and also "five" as ideas and as verbal usage can also be noted. Some few observations can also be made on the aptitude at imitating groupings of fingers or of objects where the number factor plays a part. Finally, we detect certain steps in the development of the aptitude to divide continuous quantities, to decompose the first numbers, odd or even.

Miss Drummond tells of signs of the early appearance of an interest in number, gives evidence of the slow development of understanding, tells of the persistence of the two- and three-year-old children in mastering number facts, and urges the parent or teacher to guide, not to instruct, in the early stages of number learning.[8]

Mlle. Descoeudres reports simple number tests given to 300 Swiss children whose ages ranged from two and a half to five years. She found the youngest children able to recognize small groups of objects and to count by rote. She found other number abilities appearing later, and she also found that the abilities shown early progressed as the ages advanced.[9]

Beckmann tested 465 German children, whose ages ranged from two and a half to six years and recorded ability to recognize groups of two objects present at two years and well-developed counting and assured familiarity with addition evident at six years.[10]

All these authorities agree in presenting evidence that simple number facts may be understood by children at an early age, that this knowledge steadily increases, and that there are individual variations in the depths of understanding held by different children at each age. Miss Drummond says that "the realization of the nature of number comes slowly to the child through his own activity in counting. . . . He likes to count lots and lots and lots of things. While he is doing this, he is not only gaining the abstract idea of number, he is really learning addition, subtraction, multiplication, and division, for these are all implicit in the counting."[11] Mlle. Monchamps says that some children "before entering school possess ideas of number which have already demanded mental effort of greater intensity than any later mathematical exercise will ever necessitate."[12]

In one way, number as a school subject differs materially from the other school subjects: the first important steps are taken by the learner himself without the aid of a teacher. Miss Drummond has pointed out six somewhat overlapping stages[13] in early counting which the teacher should understand clearly:

1. The pre-number stage. The child has no real idea of counting or of number at all.
2. The child begins to know *one*. "Two" and sometimes other number terms are used as indicating *more than one*.
3. Recognition of small groups. Knowledge of the order of the first four or five number names. Number names may be used to denote magnitude.
4. Knowledge of the number series up to ten or even twenty. The counting of small groups of objects is *sometimes* accurate.
5. Increasing familiarity with the number series. Some hesitation at the tens. Counting

[7] Decroly and Degand, *op. cit.*, pp. 118–119.
[8] Drummond, Margaret. *Psychology and Teaching of Number*. Yonkers, New York: World Book Company, 1922.
[9] Descoeudres, Alice. *Le Developpement de l'Enfant de Deux à Sept Ans*. pp. 233–82.
[10] Beckmann, Hermann. "Die Entwicklung der Zahlleistung bei 2–6 jährigen Kinders." *Zeitschrift für Angewandte Psychologie*, 22: 2–72, 1923.
[11] Drummond, *op. cit.*, p. 12.
[12] Quoted in Descoeudres, *op. cit.*, p. 237.
[13] Drummond, *op. cit.*, p. 35.

of small groups of things extremely careful and accurate.

6. Perfect familiarity with the number series. Ability to count any number of things. Mistakes in counting may now be set down to inattention. They are on a par with mistakes made by adults.

Only ten of the thirty-five children in the imaginary first grade have reached the fifth stage. There is a possibility that three of them, the children who count to 100, are in the sixth stage. Miss Drummond's caution[14] against disregarding the stages is of value to the teacher:

> Until the child has reached at least the fifth stage, formal teaching is likely to do more harm than good. It is apt to make the child think of number as a difficult subject, and the inner resistance which is thus produced accounts for many, if not all, of the people who never feel at home with arithmetic and mathematics. On entering our schools, children may be at any of these stages. Hence no class teaching of number should be attempted in the Infant Department. Whatever may be said of the respective merits of individual work and class work in other subjects, it is quite certain that all early number work should be individual. It is also certain that it should be spontaneous. No normal child should be compelled to do counting every day; all normal children should be allowed to work at the subject as they usually tend to—that is, they should have facilities for doing periods of intensive work and for leaving the subject altogether aside.

The teacher's task with our imaginary first grade is to provide activities by which the 25 children who are below Miss Drummond's fifth stage in counting may eventually reach that stage. She has a difficult problem, for the learning must be individual; it must be through experience and not by instruction; it must progress at the child's own rate, and each step in advance he must take when he is ready for it. On the teacher's part this will require a sympathetic understanding of the signs of number familiarity. She will have to know the meaning of experiences like this: John's teacher had asked him how much 4+1 were. He could not tell her, so she gave him 4 books and then a single book to count. He was able to give her a satisfactory answer, but when she said, "Now how many are 4+1?" he started again at 1 and counted to 5. To John 4 was not yet an entity; it was a series of individual objects which must be enumerated. Although John was in the third grade, he was in the earliest stage of number understanding. His teacher saw his difficulty; she gave him frequent opportunities to count paint pans, pencils, books, sticks, beads, and the like, and allowed him to work by himself. He became so interested that he came in before school and stayed in at recess time to work with numbers. Soon he was learning addition. Within six weeks he had overtaken the class and now he is doing satisfactory work in the fourth grade.

The first-grade teacher will also need to appreciate the tiny increments of understanding by which a young child's knowledge of number grows. She must realize that she can help most by answering questions, by providing experiences, by arranging activities, and by offering suggestions of additional facts to be learned in a situation.

[14] Drummond, *ibid.* p. 35 ff. The schools referred to are Scottish schools.

Number Ability of Preschool Children

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THE present discussion reports an investigation whose purpose was (1) to analyze in preschool children the development of three phases of quantitative experience, namely, counting, recognition of number aggregates, and combination of aggregates; and (2) to determine to what extent the development of ability to deal with number is conditioned by general intelligence.

NATURE OF THE INVESTIGATION

These problems were studied by means of two tests given to 125 children ranging in age from 36 to 72 months. One was a test of number ability and the other a test of general intelligence. All children tested were enrolled in nursery schools or kindergartens and were so selected as to form three age-groups of comparable I.Q. The use of age-groups of approximately the same I.Q. made it possible to compare attainment at successive age-levels. Comparison under these conditions assumed that three-year-olds would attain approximately the same development a year later as that of the four-year group of comparable intelligence. Similarly the attainment of the five-year group was assumed to be that which the three-year group would reach two years later. Conditions of scientific exactness would doubtless have been met more adequately if the measures obtained were the results of annual retests of the same children. Such a procedure, however preferable, was out of the question with children of these ages due to the numerous factors affecting continuous enrollment in the same schools.

The place for testing was usually a small room in close proximity to the regular classroom. The equipment included a low table and a couple of kindergarten chairs of standard heights. A dark oil cloth covering for the table showed to advantage the light colored material used in the various test-exercises. The attitude of the children toward the test was consistently one of play. They enjoyed manipulating the materials and the boxes in which they were kept. Some, Pandora-like in their curiosity, peeped under the lids to discover the contents. Not one of the children evidenced any realization that the "games" were test-exercises. The attitude toward the examiner was friendly and cooperative. After the first visit or two to the school her appearance was marked by a cordial welcome that not infrequently included an insistent invitation to "join in" in their play or the birthday party for Jane or Bobby.

THE TESTS

The number test was administered individually to each child in the three age-groups. It was arranged in three series. Series one contained three exercises in counting; series two, four exercises in recognizing small number aggregates; and series three, eight exercises in combining numbers under varying conditions. The time required for the test was approximately fifteen minutes and the length of interval between the first and second trials was about ten days.

The purpose of the first counting exercise was to determine how far the child could repeat correctly by rote the terms in the number series. The examiner said, "Let me hear how far you can count. Begin counting in this way: 1, 2, 3, and go as far as you can." The aim of the second exercise was to secure a measure of the child's ability to employ these terms in counting twenty or more small cubes arranged in parallel rows. In this exercise the examiner said, "Count these little blocks in this way: 1, 2, 3, and so on. Touch each one as you count it." In the discussion that follows, this ability is called rational counting. The third exercise sought a measure of ability to count backwards from twenty. In the Stanford Revision of the Binet-Simon Scale this is one of the commissions in the test for eight-year-olds.

The median intelligent quotient for the three successive age groups was 110. Average scores for those below and those above this median and for the group as a whole appear in the first, second and third columns respectively of the three sections devoted to the age-groups in Table I.

TABLE I

SCORES FOR COUNTING EXERCISES IN SERIES I OF NUMBER TEST

Counting Exercise	36–48 months			48–60 months			60–72 months		
	Below Md. I.Q.	Above Md. I.Q.	All	Below Md. I.Q.	Above Md. I.Q.	All	Below Md. I.Q.	Above Md. I.Q.	All
1. Rote	3.0	5.8	4.5	12.0	24.0	17.6	23.7	43.2	33.4
2. Rational	3.2	5.3	4.4	10.3	19.1	14.5	21.6	34.5	28.2
3. Backwards	0.0	0.0	0.0	0.0	1.6	.7	4.4	6.5	5.5

RESULTS IN COUNTING—SERIES I

Age-Group Variations. Scores for all three phases of counting showed appreciable progress from age-group to age-group. In rote counting the range is from 4.5 at three years to 33.4 at five. There was a rapid advance from the fourth year to the fifth year due to greater control of numerical terms. Ability in rational counting was found to be slightly less than that in rote counting. While this result is in agreement with recent findings for slightly older children, earlier investigators reported that memorization of number names as used in rote counting proceeds far in advance of ability to use these in enumerating objects in a series. Ability in counting backwards did not appear before the fourth year and then among the brighter children only. In every instance successful achievement in this skill was accompanied by high scores in other kinds of counting. Development continued in the fifth year though the score indicated little facility for the group as a whole.

Relation to I.Q. Scores for higher and lower intelligence groups showed that superior attainment accompanied superior I.Q. The higher I.Q. groups at each age level approximately doubled the scores of corresponding lower groups in rote and rational counting.

Study of Individual Records—Rote Counting. Analyses of the children's individual records furnished further insight into problems of growth in number ability. All three-year-olds counted by rote to 2, and 69.2 percent counted to 4. One child counted to 12. Among four-year-olds 51 percent counted beyond 12; 18 percent reached 20; and 12 percent went beyond this point. On the other hand, 9 percent failed to reach 8, an achievement inferior to that of the best among the three-year group. Ability in the third year was small in amount and confined within narrow limits; that in the fourth year was not only greater in amount but the "spread" was markedly wider, thus indicating increased variation in the function measured and a tendency toward retardation.

Rote counting is more conspicuously developed in the fifth than in the two earlier years. Fifty-five percent of the group reached the second decade; of these 22.5 percent counted several decades beyond, and 12.5 percent counted to 100. A lesser achievement was made by 37.5 percent who bridged the first decade but failed to reach the second. One child did not progress beyond 8, a score surpassed by two children in third, and by all but four children in fourth year. Though an eighth of the group reached 100 and the entire group reached an average of 33.4,[1] one

[1] This average surpasses those reported in the studies made at the Child Welfare Station, at the University of Iowa, by Buckingham and MacLatchy. This discrepancy is probably best explained as due to a higher selection of children and their classification into comparable I.Q. groups in this study.

child had not yet mastered the problem of serial order of numerals.

Rational Counting. The lesser achievement in rational as compared with rote counting, though small, may be explained for preschool children by the greater complexity of the process involved in enumerating objects in a series. Rote counting employs simple recall of verbal numerals in correct serial order, but rational counting uses this function and besides, requires the matching or tallying of the correct verbal numerals with the corresponding objects being discriminated. That is, rational counting employs a relational form of thinking which holds the two factors, number names and discriminated objects, together in a one-to-one relation. The advanced character of this type of thinking has been well discussed by Royce.[2]

The idea of series which must be assumed as basic to rational counting, Royce points out, is the result of iterative, recurrent thinking that applies the definite first unit discriminated to successive units in the objective series, but the relationship is kept constantly in mind. . . . He adds, "Such processes of passing to 'one more' instance of a given type, are processes not of barren repetition, but of genuine progress to higher stages of knowledge. . . . We can keep *tally* of our objects only if we combine the successive series of acts of observing another and yet another object in our collection of objects with the constant use of the already ordered series of number names, whose value depends upon the fact that one of them comes *first*, another *second*, etc., and that we well know what this order means."

Rational counting is thus seen to involve complex mental activities. A few two-year-old children were tested to determine to what extent discrimination of objects in a series was observable. One child, twenty-seven months old, pointed to the separate items arranged in a row saying as he touched each, "Dee-dee-dee"; three others a few months older used the terms, dee-one, dee-one, dee-one; or dat-one, dat-one, dat-one; or one, one, one.

[2] Royce, Josiah. "The World and the Individual," Supplementary Essay: The One, the Many and the Infinite, *First Series: Gifford Lectures*. New York: The Macmillan Company, 1901. p. 529 ff.

This immature form of response is replaced later, as was shown on the individual records of the three-year-old children, by the first few terms of the numeral series. Inability to master the idea of correspondence limits rational counting for three-year-olds to approximately the first decade; 61 percent of the group reached 4 and one child counted to 11. In the fourth year two-thirds, and in the fifth all but one child bridged the first decade; 15.1 percent of the fourth and 47.5 of the fifth-year group counted beyond 20; a few reached 100. Throughout the three successive years there was a steady advance in ability to enumerate correctly the items in the concrete series, though in general, achievement in rational counting was less than that in rote until the first three or four decades were reached.[3]

Recognition of Group or Aggregate Number—Series II

Four exercises were used to determine to what extent the ability to recognize groups was developed in preschool years. Each exercise measured one element of recognition as follows: (1) matching a collection of concrete items with a like numbered collection; (2) discriminating larger from slightly smaller groups; (3) forming groups on request; and (4) naming group numbers as seen on flash cards. The aim in all four exercises was twofold, namely, to determine the extent to which the response was to the numerical size of the group and the methods which were used in making the response.

Age-Group Variations. Scores for each of the separate exercises indicated that progress from year to year was steady but greater in some elements than in others. For example, forming groups is an easier process than naming groups shown on flash cards; the latter ability is practically at zero for the youngest children. Throughout the four exercises the records for the higher I.Q. groups were considerably greater than those for lower I.Q. groups.

[3] Binet, Alfred and Simon, Thomas. "The Development of Intelligence in Children," (Trans. by E. Kite), *Training School Bulletin*, Vineland, New Jersey, 1916.

It is interesting to note in this connection that in the last revision of the Binet-Simon Scale the collaborators, who considered a child's facility in counting as one measure of mental maturity, declared, "A three-year-old child cannot count four sous; at four years half the children succeed; at five only retarded children fail."

The method usually employed to determine the number of objects in a group was to count them. At the three-year level counting ability is not far advanced, as was pointed out in the preceding analysis, but with the development of the idea of one-to-one correspondence in the succeeding years counting ability improves rapidly. Hence, the older group showed marked improvement in forming number aggregates by the counting method.

Analysis of Methods. Responses to the four exercises disclose wide variation in methods employed by different children. Good "counters" used mature methods, while those who had not yet grasped the idea of correspondence used immature methods. It was found that the individual's way of responding gave more insight into the extent to which he perceived groups than did the individual scores obtained. For example, two children receiving identical scores often arrived at their responses by very different modes of thinking that signified a wide difference in real maturity in number ability.

The methods of response showed advance in maturity from age to age. All but one of the three-year-olds sensed the problem of matching; some matched 2 and 3 but laid down handfuls for larger numbers to indicate their idea of "many" and hence had too many objects to count. Brighter ones identified groups by counting but for them "how many" was still the counting of separate unities. One child had advanced to the point where he could recognize 2 and 3 without counting, and 4 as 2 twos.

Among four-year-olds the idea of "many" was designated by a favorite number, ten-two, twenty-ten and the like; 2 and 3 were readily recognized without counting; larger numbers were attacked at once by counting or were broken up into twos, threes, or, more rarely, into fours or fives; only the brightest three-year-olds saw counting as a means of matching, forming, discriminating or naming number groups while few four-year-olds failed to realize this fact; one subject who ranked highest in total score and was fifth from highest in general intelligence responded to 3, 4, and 5 by beginning with 2 and counting to the remainder one at a time.

Practically all five-year-olds recognized 2 and 3 at a glance; 4 was often so recognized, but to many it was 2 twos or "chee" and one more; high scores were made by good "counters." In the flash card exercise, one child saw 8 as 4 and 4, but closing her eyes she counted on from 4 to 8 to check her response. Similarly, 9 was seen as equivalent to 5 and 4, but the total was checked by counting to 8 by twos and adding on 1. One boy who readily recognized aggregates from 2 to 9 declared he could form 8 in "lots of ways" and, using the small cubes of the exercises, he showed that 4 and 4 made 8, 6 and 2, 5 and 3, etc. In each case he explained how he took units out of one 4 to build up the other 4 to a 6, 5, or 7.

Significance of Method. The foregoing summary makes clear that the order of development is from perception of simple spatial forms of aggregates toward analysis, first by counting single unities, later, by recognition of small numbers as 2 or 3 and counting on the other unities in the group number. Still later these small groups may be combined by counting by twos, or by combining "doubles", i.e. 2 and 2; 3 and 3, etc. Finally, a mature stage is reached, characterized by the prompt recognition and naming of aggregates as group or cardinal numbers. Maturity of method may vary from exercise to exercise with the same child, but in general more mature methods are closely associated with superior achievement in rational thinking.

The awareness of cardinal number involves a higher stage of abstraction than is the case in serial counting. The child who has achieved this goal in number knows it as a new kind of reality. One writer[4] points out:

> A group of five things is a reality distinct from the individual objects which compose the group. Five people are not merely five separate experiences; they are a group. The group has a kind of reality which is broader and more inclusive than the individuals of which it is composed.

[4] Judd, Charles H. "Psychological Analysis of the Fundamentals of Arithmetic," *Supplementary Educational Monographs*, No. 32. Chicago: Department of Education, University of Chicago, 1927, p. 108.

Combining Aggregates of Two or Three Numbers—Series III

Combining was a difficult task for all the children tested. Few three-year-olds had any understanding of the task. A few who attempted it employed counting methods. A year later counting became the general procedure, though a few of the four-year group recognized small aggregates from which they counted forward.

Mature five-year-olds grasped the idea of augmentation more clearly as that of counting on from a familiar small number. A few counted forward by twos to a point of vantage and continued by ones to reach the sum sought. One or two children who knew a few simple combinations used these. The least mature children who continued the tallying methods of the younger years used such concrete methods as bobbing their heads, tapping with fingers or feet or the like. In some instances the use of tallies had become a deft skill not easily detected while in progress.

Of the three six-year-old children, as with older subjects, it may be said, "There is . . . a vast difference between being able, on the one hand, to recognize familiar groups and being able, on the other hand, to perform the elaborate mental act of explicitly defining the number in such a group."[5]

Interdependence of the Three Processes

The detailed analysis of counting, recognition of aggregates, and combining aggregates make evident certain generalizations. Rote counting has been shown to develop just slightly in advance of rational counting. It involves the memorization and accurate recall of a fixed order of numeral terms. Rational counting is a complex mental process dependent upon grasping the idea of one-to-one relation between these numeral terms and the items discriminated in an objective series.

Characteristic errors in the early stages of counting include failure to recall correctly the number terms, confusion in matching terms with the objects being enumerated, and inability to keep the place in either series.

Recognition of aggregates, though facilitated by ability in rational counting, involves more complex mental processes. Group recognition with the youngest subjects did not, except in a few cases, extend beyond two, but with the older and brighter subjects it extended to three, four, and, in some few cases, to five. The more mature children by numerous methods and insights showed that they regarded the group as having a distinct entity of its own dependent upon, but not the same as, that of its several constituent items. This is a new relational factor as significant for later development as the principle of *correspondence* is in counting. Methods that tend to make this new relational factor articulate, employ matching of small groups in pairs or doubles; recognition of aggregates by general form, symmetry, or familiar analogies; extension of the pairing function into rational counting by twos or larger units; a breaking up of larger groups into smaller ones readily counted or "seen." These and other insights aid the singling out of a new factor that in turn becomes an essential factor in the growth of ability to combine aggregates.

Among three-to six-year-old children the process of combining has not advanced very far. Few of the youngest even sense the task. For the four-year-old it is largely a process of counting. Competency is distinctly greater with two than with three numbers. This may be explained, in part, by the greater complexity in the mental process involved in holding in mind the partial sum until the third number is added to obtain the total sum. Ability in the combining function differs with different types of experience. Combining visible objects is much less difficult as a process than combining the same number of objects seen in timed exposure, or as imagined, or as abstract numbers. The obvious explanation offers that in the first situation the rational counting facilitates the process more readily.

Growth in number ability at these immature ages within the limits of the study is very gradual and dependent upon many

[5] Judd, Charles H. *op. cit.*, p. 99.

factors. Brightness is one such factor that strongly influences superior ability at each age level. Furthermore, progress is intimately interrelated and dependent upon mastery of more mature methods of dealing with number experiences. These more mature methods are evolved as the child gets insight or understanding of the relational factors existing between numbers.

Practical Values

The study yields certain practical values:

1. To students of mental development, particularly teachers and parents of young children, it offers objectively derived inventories of abilities to deal with number ideas at early age levels. Such inventories help to supply a scientific basis for teaching and learning in grades one and two.
2. It presents an analysis of different methods employed by children prior to school instruction in dealing with three number processes.
3. It indicates types of difficulty encountered in number learning at early stages. Other investigators report that early difficulties persist and become causal factors of retardation in arithmetic, for achievement in arithmetic is dependent upon a hierarchy of habits which depend upon each other in sequence.
4. It presents the methods of generalization developed by preschool children in dealing with number. The dependence of number ideas upon ability in generalization and abstract thinking has been emphasized by Royce, Stern and Judd, but little objective data are known for early ages.
5. It shows the correlation between ability to deal with number and general intelligence.
6. It defines one type of test which should be used in determining mental maturity for first-grade entrants.

From *Sung Under the Silver Umbrella* *Courtesy of the Macmillan Company*
Illustrated by *Dorothy Lathrop*

Ships go so many places,
I think most seventeen lands.
How far can your ship sail?

Young Children and Fractions

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CHILDREN in the primary school know a great deal more about fractions than is often imagined. A study which justifies this statement was made with 266 children from the kindergarten, the first, the second, and the third grades of the Elementary School of the University of Chicago.[1] Each of these children was given an individual test. The purpose of this individual testing program was to discover:

1. Whether primary school children have certain concepts of fractions.
2. If so, what their concepts are.
3. When the children acquire these concepts.
4. How they acquire them.

There was, at the time of this study, no teaching of fractions in the primary grades of the school where this study was carried on. Therefore, through the formulation and the administration of a comprehensive series of tests, it was possible to determine what the children had learned from their everyday experiences about fractions.

DESCRIPTION OF TESTS

Concepts Tested. The tests were designed to measure the extent to which the children had acquired several different concepts of fractions: unit fractions as $\frac{1}{2}$ of a single thing, $\frac{1}{4}$ of a single thing, $\frac{1}{2}$ of two things and of four things, $\frac{1}{4}$ of four things and of eight things, etc.; proper fractions that are not unit fractions as $\frac{3}{4}$ of one thing, $\frac{3}{4}$ of four things, etc.; improper fractions; the identification of fractions; and equivalent fractions. In each test there was opportunity for objective expression as well as for numerical expression. For example, the child was given a square and a pair of scissors and he was asked to give the examiner a half of the square. In the next part of the test he was shown a square that had been cut into halves and he was asked to tell what he would call each piece.

[1] "The Concepts of Fractions of Children in the Primary School," unpublished Master's Thesis by the writer. Department of Education, University of Chicago.

Test Situation. Each test was made up of several exercises and each exercise was composed of several items. So the children had many opportunities to demonstrate whether they knew anything about the subject upon which they were being tested. In all, each child had forty-two opportunities to respond either objectively or numerically. The first items were very easy so that the children could be successful. The materials used were bright in color, interesting to the children because they were within their experience, and presented as attractively as possible in a very pleasant room. The children enjoyed the experience. They told each other about the games which they had had an opportunity to play and consequently when the examiner went to the class rooms for the children they begged to be chosen. When the examiner asked them what they knew about the tests they were to take, it was clear that they knew very little except that they were "fun." In no case did the examiner feel that a child's responses had been affected by talking with other children. But in almost every case the examiner felt that the children had come to the test with greater pleasure and satisfaction on account of what they had heard.

Sample Items. It is impossible to record here all of the items which made up the five tests. But a few examples are given:

$\frac{1}{2}$ of 2. The examiner places two pencils on the table and says, "Here are some pencils. Will you give me one-half of them?"

$\frac{1}{2}$ of 4. The examiner places four pennies in a row and says, "Here are some pennies. I am going to pick up some of them. (She takes away one penny.) Have I picked up one-half, one-fourth, or one-third of these pennies?"

One thing is $\frac{1}{2}$ as much as the other. Place a paper in front of the child and say, "I am going to draw a line for you. (Draw a line about six inches long.) Now will you draw a line for me that is one-half as long as mine?"

One thing is $\frac{1}{3}$ as much as another. Give the child two sticks of artificial candy. One is one-

third as long as the other. Then say, "Here are two sticks of candy. Is the little stick one-half, one-third, or one-fourth as long as the big one?"

Two things are $\frac{1}{2}$ of four things. Show the child a picture of four colored apples in a row. Say, "Here are some apples. Will you draw for me one-half as many apples as I have drawn."

$\frac{3}{4}$ of a single thing. Give the child a paper with the picture of a drinking glass drawn on it. Say, "Here is a glass. Will you color three-fourths of it to show that it is three-fourths full of orange juice."

$\frac{3}{4}$ of four things. Give the child a paper with four small circles drawn on it. Say, "Here are some marbles. Will you color three-fourths of them."

"You know, don't you, that one is the same as how many halves? You know that one is the same as how many thirds? You know that one is the same as how many fourths?"

"Will you tell me what is the same as three-halves, as four-halves, as four-thirds, as five-fourths, as seven-fifths?"

Difficulty of Tests. The separate tests were arranged in what was supposed to be the increasing order of their difficulty. The items, however, were not graded. There was no experimental check on the difficulty ranking of the tests, of the exercises, and of the items until the children began to take the tests. Then it was found that no kindergarten child was able to respond satisfactorily to any of the tests except the first, that first-grade children went a bit further, and that second-grade children went further still, while the third-grade children responded satisfactorily to three or four tests.

Out of forty-two opportunities the kindergarten children averaged 4.9 satisfactory responses, the first grade 6, the second grade 11, and the third grade 20.3. In no case was a child permitted to become discouraged. When he felt he could not respond to an exercise he was not urged to do so. And when the examiner saw that the work was clearly beyond a child she stopped the testing.

A few children were anxious to know whether their responses were satisfactory or not. In order to meet this situation the examiner expressed frequent satisfaction with the child's work by saying, "That is fine." "You are a good worker." "I like the way you work." As a result many children were happy over responses that were incorrect. They had pleased the examiner and that was enough. Their worries vanished when the examiner directed their attention to the way they worked rather than to the correctness of their work.

Recording of Responses. Two types of responses were noted and recorded, the children's spoken answers or explanations and the accompanying behavior. All answers were recorded verbatim, and all visibly important accompanying responses were scrutinized. Answers and significant behavioral responses were filed and recorded in two forms (a) as descriptive statements or (b) as satisfactory or unsatisfactory responses to be recorded later in the form of tables.

The judgment as to whether a response was satisfactory or unsatisfactory was made in this way. If a child's response was correct and he demonstrated by his actions, or explained with words, why he knew he was right, his response was counted as satisfactory. For example, the child was asked to make a picture of a glass look as if it were filled three-fourths full of orange juice. Now, if he estimated four equal parts with his fingers or with his crayon along the edge of the glass and then filled in three of those parts, his behavior was counted as evidence of his understanding. On the other hand, if the child seemed to guess and gave no correct indication in his behavior, he was asked to explain how he knew that the glass was three-fourths full. If then his explanation was satisfactory, he was given credit for his response. In all cases of course the child's judgments were expected to be approximate and not exact.

After the tests had been given and the responses had been recorded in the form of descriptive statements and tables, both were examined for answers to the four questions raised as problems for the investigation. The results of this study are accordingly reported in four sections.

Results Obtained

1. *These primary school children had certain concepts of fractions.*

Not all of the evidence can be presented here but from the descriptive statements these examples have been selected:

A child was given a picture of six jars of vegetables on a shelf. Four of the jars contained green vegetables and two contained red vegetables. After talking about the vegetables, the examiner said, "What part of these vegetables is green?" And the child replied, "Four-sixths . . . four-sixths. What do you call those things that are second from thirds? I think they are sixths." Such a remark as this demonstrates that the child understood what he was talking about.

The examiner held up four pencils and said to the child, "Here are some pencils. Will you take one-half of them?" The child took two and answered, "Two." When asked why he knew that two were a half he explained, "In between two and two there is a middle, and here is the middle." As he talked he grouped the pencils in twos and placed his finger in between the two groups. Here again is evidence of perfect understanding with illuminating examples.

From the tables originally constructed, one has been selected as evidence that the children did have certain concepts of fractions. In this table, Table I, *R* refers to satisfactory responses and *Ch.* refers to the number of children.

This table should be read: one satisfactory response was made by two children, two satisfactory responses were made by seven children . . . and forty satisfactory responses were made by two children. The important observation here is that every child gave evidence that he knew something about fractions.

TABLE I

NUMBER OF SATISFACTORY RESPONSES CHILDREN WERE ABLE TO MAKE TO THE TESTS ON THEIR CONCEPTS OF FRACTIONS

R.	Ch.	R.	Ch.	R.	Ch.	R.	Ch.	R.	Ch.	R.	Ch.
1	2	8	26	15	8	22	2	29	2	36	2
2	7	9	19	16	3	23	2	30	2	37	3
3	7	10	11	17	4	24	1	31	0	38	3
4	16	11	7	18	13	25	0	32	1	39	0
5	34	12	4	19	0	26	0	33	2	40	2
6	20	13	7	20	0	27	1	34	2	41	1
7	24	14	6	21	3	28	1	35	0	42	0
Total										42	248

2. *What concepts of fractions the children had.* The best evidence of this is found, not in the descriptive statements, but in Table II.

From this table it is seen that the children who took the tests understood unit fractions better than any other fractions included in

TABLE II

CHILDREN'S RESPONSES TO EACH TEST

Test	Concepts Tested	No. of Resp.	Percent of Total Resp.	No. of Sat. Resp.	Percent of Sat. Resp.
I	Unit Fractions	3968	.43	2117	.53
II	Other Prop. Fractions	2197	.24	394	.18
III	Improper Fractions	1512	.17	135	.08
IV	Identif. of Fractions	1134	.12	99	.08
V	Equiv. Fractions	387	.04	1	0
	Total	9198	1.00	2746	

the test; that these children knew something about proper fractions which are not unit fractions; that very few of the children knew anything about improper fractions and about the identification of fractions; and that only one of these primary school children knew anything about equivalent fractions.

b. They knew a unit fraction when it was used in the comparison of two objects. "This is ½ as big as that," etc.

c. The children knew less about the unit fraction as applied to a group of objects than about the unit fraction as applied to a single object. ½ of 1 is much easier than ½ of 4.

d. The children could use fractions in comparing two objects better than in comparing two groups of objects. "This is ½ of that" is easier than "3 is ½ of 6."

TABLE III

AVERAGE NUMBER OF SATISFACTORY RESPONSES MADE BY EACH GRADE, EACH CHRONOLOGICAL AGE GROUP, AND EACH MENTAL AGE GROUP TO EACH OF THE TESTS, I THROUGH V

	Concepts by Tests	Grade Groups			
		Grade III	Grade II	Grade I	Kgn.
I	Unit Fractions	11.9	9.4	5.9	3.7
II	Other Proper Fractions	3.8	1.7	0	0
III	Improper Fractions	1.2	.4	0	0
IV	Identification of Fractions	1.6	.1	0	0
V	Equivalent Fractions	0	0	0	0
		M.A. Groups			
		11–13	9–11	7–9	5–7
I	Unit Fractions	14.5	11.1	7.0	4.5
II	Other Proper Fractions	7.0	2.8	.5	0
III	Improper Fractions	3.5	.9	0	0
IV	Identification of Fractions	2.8	.5	0	0
V	Equivalent Fractions	0	0	0	0
		C.A. Groups			
		10–12	8–10	6–8	4–6
I	Unit Fractions	12.0	9.2	8.7	4.6
II	Other Proper Fractions	5.3	2.5	.8	0
III	Improper Fractions	3.7	.9	.2	0
IV	Identification of Fractions	2.3	.7	0	0
V	Equivalent Fractions	0	0	0	0

These tests demonstrated, too, that children knew much more about certain fractions than about others. From this study it was found that the children knew the fractions used in this study in this order: 1/2, 1/4, 2/4, 1/3, 3/4, 2/3, 4/3, 3/2, 3/5, 7/5, 5/4, 2/5.

These tests also demonstrated:

a. That the children knew more about a unit fraction as applied to a single object than about any other of the fractions used in the test: ½ of 1, etc.

3. *When children acquire their concepts of fractions.* The most consistent gain in the acquisition of fraction concepts is evident when the responses are arranged in accordance with the chronological ages of the children, as in Table III above.

From this table it is seen that the group of four- to six-year-olds had some concepts of unit fractions; that some of the children

began to understand proper fractions other than unit fractions and even improper fractions between the sixth and eighth years; that these children began to identify fractions between the eight and tenth years; and that they were almost wholly ignorant of the concept of equivalent fractions.

4. *How these children gained their concepts of fractions.* The descriptive statements which contain the children's verbatim responses give some indication of the ways in which they acquire their fraction concepts. Unfortunately only a few of the children's statements can be given here as they are most enlightening. From the many, these are chosen:

A four-year-old said, "Every day after I have my nap, Mother gives Brother and me an orange. Sometimes I cut it and sometimes he cuts it. That's how I learned one-half."

A first-grade boy said, "I learned quarters from waffles. There are lines on the quarters. I asked mother what they were for. She said they were quarters and I remembered that."

A second-grade child said, "I don't know exactly how I learned some of them. I have always known a half. Some of the others I have figured out for myself. I like to figure out things like that. I know that a quarter is a half of a half, and lots of other things like that."

A little girl said, "I have two sisters. Mother often cuts things for the three of us. That is how I learned thirds. I don't know how I learned halves and quarters."

Another said, "In school we often fold halves and quarters in the drawing class. That is how I learned them."

Significance of the Study

Such statements reveal that these children gained much knowledge of fractions from their experiences at home and at school, that is, from their daily living. It is true that the data reported were obtained from only 266 children and that these children may well have been a selected group. On the other hand, since these 266 children did learn about fractions, of their own volition and from their ordinary experiences in living, it seems reasonable to believe that other children must be learning much about fractions in just the same way. It seems reasonable, too, that if primary children learn so much about fractions without systematic instruction of any sort, there can be no question about their ability to learn more about fractions under direct teaching. The inference is that primary children should be exposed to the kinds of experience which bring about learnings in the group and that the primary school curriculum in number should contain opportunities for children to acquire some of the simpler fundamental ideas about fractions.

This study demonstrates that it is safe to include in the primary school curriculum such concepts of fractions as these:

For the first grade: (a) The concept of unit fractions, when the fraction is applied first to a single object and then to a group of objects; (b) the concept of unit fractions when used in the comparison of two objects, and later of groups of objects.

For the second grade: (a) the concepts listed above, and in addition, (b) the concepts of proper fractions other than unit fractions and (c) a beginning in the understanding of improper fractions.

For the third grade: (a) the concepts listed for the first and the second-grade children and in addition, (b) the identification of fractions.

The study also demonstrates that the teaching of fractions should be in the form of concrete experiences with real objects since this was the way in which the children tested had gained their understanding of fractions.

The study demonstrates the need for a program of experimental teaching of fractions in the primary school in order to determine what the children can learn about fractions as a result of teaching; what materials best lend themselves to the teaching of fractions in the primary school; and what experiences provide the most abundant and the most valuable opportunities for acquiring the concepts that are attainable at this level.

Materials and Methods

EDWINA DEANS
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THIS article represents an attempt to give some practical suggestions with regard to teaching number in the primary grades. The materials and methods to be described have proved their worth under the test of classroom use. They were originally developed in response to a belief that, properly taught, arithmetic should be to the primary child a series of happy, fascinating, yet educative experiences. In the school where these materials and methods were devised and tried, arithmetic has become just such a series of experiences. The children look forward eagerly to the number period; they enjoy every minute of it; and best of all, they learn arithmetic.

BASIC THEORY

It is frequently stated that the purpose of arithmetic in the elementary curriculum is to develop expertness in quantitative thinking. This expertness is not to be had by merely hoping for it. On the contrary, it must be made the object of direct instruction. The child can learn to think quantitatively only by engaging in quantitative thinking. As part of his equipment he must possess a fund of rich number meanings. Nor are these to be had merely for the hopeful wish. Instead, they must be taught, for they come only as the result of definite reactions on the part of the child to number as presented in a wide variety of situations.

It is possible for any teacher to secure much light on the way in which children develop sound number concepts. Let her but study a group of children individually to discover how they think of the number combinations, for example. The difficulties they encounter, the methods they build up for themselves, the tenacity with which they hold to these methods—all demonstrate that number concepts are slow in their development and are by no means easy to acquire. Number concepts as we know them do not happen all at once; rather they evolve, step by step, stage by stage, each stage in turn serving the purpose of lifting the child's thinking to more mature levels. If these stages are to be in reality increasingly more mature, activities must be carefully selected and ordered with this end in view.

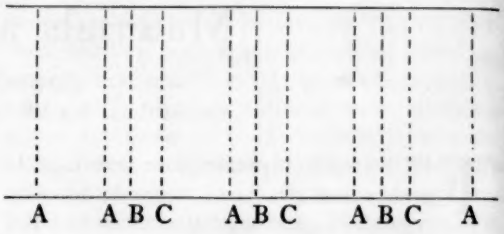
It is not implied that activities to be useful must be artificial and unnatural. Quite the contrary is true. In the first place, all planned activities should be presented as normal happenings. In the second place, the teacher should recognize the possibilities for extending the meaning of number in the daily environment and conduct of the school child. She should avail herself of every opportunity which arises to point out the usefulness of number, thus stressing in a natural way its social significance. Nevertheless, activities arising from this source do not give every child in the class the opportunity to "learn by doing." Each child must be assured an abundance of number experiences. On this account several very pointed problems present themselves to the teacher. They are: What materials are appropriate? How may they be obtained and taken care of? How should they be used to insure growth?

CONCRETE MATERIALS

Concrete materials are recommended for early work in number. These materials should be small enough to enable the child to handle them readily. At the beginning of instruction with concrete materials, each child should be supplied with a set of perhaps ten objects of each type decided upon. It is a practical asset if these objects have one flat surface, since this makes for ease in handling and prevents them from rolling off the table or desk. Many of these materials may be brought from home by the children themselves, or they may be purchased at a nominal cost. A list of materials of this type, with some suggestions as to the sources from which they may be obtained, is given on the page which follows:

1. Acorn caps may be collected and brought to school by the children.
2. Old buttons may also be brought from home.
3. Button molds may be purchased at a low cost. These are more satisfactory than old buttons, since they are sure to be uniform in color, size, and shape. The matter of uniformity is important, since elements of difference draw the child's attention away from number facts they are supposed to represent.
4. Colored sticks and splints of the commercial type are a part of the permanent equipment of many primary rooms. If not, they may be purchased cheaply.
5. Small black or steel shirt buttons may be obtained from any laundry. A certain grade has found these especially delightful. The children call them "little men" because they have somewhat the appearance of toy soldiers on parade.
6. Small pebbles having a flat side, grains of corn, and beans are also usable.
7. One teacher secured a supply of golf tees from a nearby factory.
8. The children may collect any number of flat sticks from the ice cream bars popular at the present time. These offer many opportunities for the concrete representation of number.

Some satisfactory arrangement for taking care of number materials is absolutely essential. It is evident that the matter of keeping materials is no little problem for the child. However, if each child is provided with a box in which to keep his number materials, this difficulty will be removed, at least partially. A very convenient one may be made from an ordinary thread box. These boxes, which may be had from dime or department stores for the asking, can be divided into three or more compartments by means of a folded strip of tagboard. Thus, the boxes are prepared to hold the different materials to be used during a given period of time. The folded tagboard is held firmly in place by pasting to the bottom and ends of the box. The illustration below shows how four compartments can be made by folding the lines marked A and C back and the lines marked B forward. The actual measurements are $11\frac{1}{2}''$ by $3\frac{1}{4}''$. This size strip of tagboard fits a box $3\frac{1}{2}''$ by $6''$.



These divided boxes, kept in the children's own desks, have proved a time-saving device since they eliminate the problem of distributing and collecting materials for each particular occasion. They also give the children a certain pride in owning and being responsible for their own materials.

Instruction in Enumeration and the Number Series

When adequate provision has been made for concrete materials, the question becomes, "How can the teacher be assured that the child's use of these materials will result in extended knowledge of number rather than in mere manipulative play?" In the first place, she must have some knowledge of the nature of number concepts and of the course of their development. Then, in the second place, she must carefully plan her program of number activities so as to promote the child's growth toward improved types of quantitative thinking.

The child's first systematic method of reacting to a number of objects quantitatively is by counting. Most children are able to count by rote when they enter school, but for many of them counting consists simply in saying the number names with no real conception of the process of enumeration. The child should be taught to count objects instead of reciting a meaningless series of number names. The ability to count not only is important in its own right, but it is fundamental to real understanding of later work in number. Some of the appropriate activities for group instruction in counting are described in the following paragraphs.

The children should begin counting only a small group of objects, more objects being added as their ability to count with understanding increases. Too many objects at the

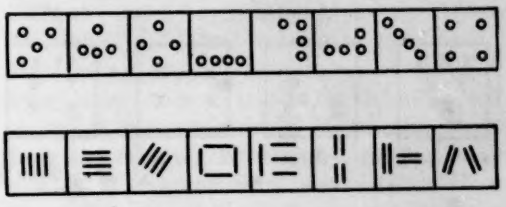
outset are confusing. Assume that opportunities have been provided for counting groups of children, of books, of desks, etc., and that instruction is to be continued now with the small objects in the boxes. The procedure with the class may be similar to the following:

"Today we are going to see how well we can count. Take out your number boxes and put them in the middle of your table out of your way. Now take out one acorn cap and put it in front of you. Take one more and place it beside the first one. How many do you have now?" When the children have the desired number of objects, they may begin over again at one, moving each object as they count it. The teacher should be sure that the children get the idea that "five," "six," and "seven" refer to whole groups of objects and not to the last-named objects in the series. The actual moving of each new object over with the group already counted helps the children to see that continued counting makes his group larger and larger.

When the children can enumerate objects correctly, work within the number series should be begun. Directions follow: "What number comes after three? What number comes before four? What number comes between three and five?", etc. One device to be used in this connection is to have six or seven children stand before the class, each child with his own number. The place of the numbers may then be dramatized.

Grouping with Real Objects

Following instruction in counting, and as a further aid to the understanding of number, the child should be given some instruction in the grouping of objects. It is desirable now that he begin to see numbers as units made up of smaller groups and as essential parts of larger groups without breaking them into ones. Counting provides the basis for developing the group notion of number, but counting will not of itself lead to the development. On the contrary, the child must be led more and more to disregard the separate elements in the group and increasingly to recognize groups immediately without recourse to counting. To develop this last ability, the child should construct groups, with concrete objects, in as many varied patterns as possible. For this purpose the larger objects in the classroom (children, pictures, windows) are ill-adapted. Much better are the child's small objects. Thus, by using acorn caps or "little men," he may construct many patterns for 4. By using sticks rather than the acorn caps he may discover a number of additional patterns.



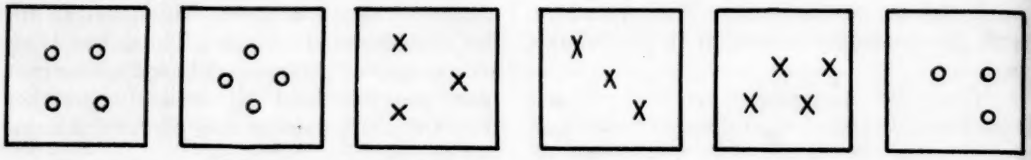
By making these groups over and over the child becomes aware of 4 as a group made up of 2 and 2, 3 and 1, and 1 and 3 rather than as a consecutive series of ones.

Grouping with Semi-Concrete Materials

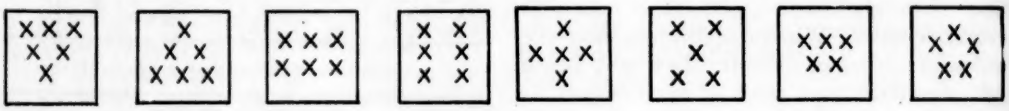
Semi-concrete number materials (as contrasted with the real objects described) are helpful in aiding the child to recognize groups readily. Cards may be made from tagboard with dots, crosses, or small pictures to represent the groups of numbers. Any or all of the groups previously constructed with concrete materials should be transferred to cards of this type. A card may be exposed to the children for a very short time, not long enough to permit them to count, and one child may be asked to give the number of objects exposed. When the patterns, as for 4, are familiar, they should very soon be mixed with patterns for 3 which the children have already learned. The illustration below shows samples of group cards for the numbers 3 and 4.

The children can probably recognize any regular pattern arrangement of groups of 3 or even 4 objects, but it is obviously impossible for them to grasp a conglomerate arrangement of 8, for instance, without counting. For this reason it is advisable for the teacher to select only regular patterns for

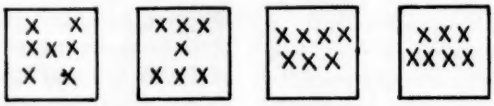
the numbers above 4. Some suggestive patterns for the larger numbers are shown in the illustrations which follow:



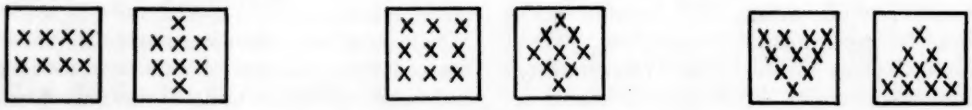
Group cards for the numbers 3 and 4



Group cards for the numbers 5 and 6



Group cards for number 7



Group cards for the numbers 8, 9, and 10

Simple Combinations in Addition and Subtraction

If the child knows the number series and can enumerate at least to 10, and if he is also able to give immediate responses to groups of objects through 5, let us say, he is now ready for a more intensive study of the number 5. (The number 5 is taken only for illustration. A beginning should be made with 4, or even 3, if necessary.) He is ready to become acquainted with the number facts involved in this number. This stage should be regarded as a period for experimentation and discovery on the part of the child. He should discover the combinations for himself through experimenting with concrete objects.

Too often a single demonstration of a number fact has been considered sufficient and has been followed at once by drill on that fact. Thus, the child has been shown that 3 blocks and 2 blocks make 5 blocks, or that 3 children and 2 children make 5 children, and immediately the number fact 3+2 has been made the object for drill and repetition. This practice is in disagreement with the plan here being described. If the number facts are to possess meaning for the child, he must see their significance again and again. Over and over he must actually "put together" objects and "take away" objects until he understands the relation of the parts and the answer. Until this stage has been attained, the attempt on the part of any child to memorize the combinations blindly should be discouraged.

Concrete Materials.—As a means of leading children to discover the addition combinations involved in the number 5, for example, the following procedure will be found of value: "Now we are ready to find out something *more* about 5. Take 5 sticks out of your

number box and lay them side by side like this:" (|||||). The teacher demonstrates at the board. "All of these sticks say 'five,' but we can make them say something else, too. Move one stick over to the left. Now you have two groups. What do these new groups say? (| ||||) Your new groups say 'one and four.' Can you make them say 'five' again? If we take our hands and push them all together they will say 'five' again, won't they? Let's do it. When we put things together we are adding." By pushing the groups together, the child gets the idea that when he adds, two smaller groups are merged into a larger group; that when he adds, he has more.

It has been found advisable to have all groups at first[1] constructed and read from left to right to avoid confusion in discussing them and to be assured that all the children are seeing the groups under consideration at the time. In like manner the remaining addition combinations in 5 may be discovered. The teacher should be sure that the groups are clearly separated. Otherwise, the children will not grasp the idea of the *two* groups which together make 5.

To supplement the above suggestions and to afford more practice in seeing the combinations, each child may be allowed to construct two other groups which make 5 and to read them to the other children. The responses will be something like these:

I have four and one are five.
Mine says two and three are five.
Mine is just like Jim's. It says four and one are five, etc.

Varying the materials will suggest a number of new ways of seeing the combinations as well as affording renewed interest on the part of the children.

The subtraction combinations should also be discovered by the use of concrete materials, and preferably at the same time the addition facts are found. That is to say, the addition and subtraction facts may well be taught together, thus revealing the meaning of addition and subtraction as opposite processes.

[1] Later on the group is "read" not only as 1+4, but also as 4+1 and as 5−1 and 5−4.

It is the general opinion of teachers that the idea of subtraction is more difficult for the child to grasp than is the idea of addition. However, much can be gained by having the child dramatize the process of subtraction by actually "taking away" objects. He must come to think of subtraction as that process by which a group of objects becomes smaller as the result of some having been withdrawn. The following is given merely as a suggestive instructional procedure:

"Take out 5 acorn caps. Make them look like this: ° ° ° ° ° (Demonstrate at the board.) How many do you have now? If we take away some, will we still have 5? Let's see if we will. Let's take away 2. Move them away from the 5-group. How many did you have at first? How many did you take away? How many do you have left? You can tell all the things you did this way, 'Five take away two are three.' Let's take away something else and see who can tell all the things you do."

There seems to be value to the child in saying either to himself or aloud the steps as he carries out the directions with actual objects—for example: "We all have 5 buttons, do we not? They look like this: ⊙ ⊙ ⊙ ⊙ ⊙. Everybody get ready to take away 1 button. As Mary takes hers away she is going to tell us what her buttons say to her." "Five buttons take away one button are four buttons." Then the steps may be reviewed as before. "How many did you have at first?", etc.

As part of the idea of subtraction the child should understand that his remainder is large or small in accordance with the number of objects taken away. If he takes away only a few objects, he has more objects left. If he takes away many objects, he has fewer objects left.

Semi-concrete Materials.—So far the explanation of work with the combinations has been concerned with real concrete materials. To prevent an abrupt break between concrete and abstract number, materials of a semi-concrete nature, similar to those advocated for instruction in grouping, have been found helpful. Materials of this type furnish additional opportunities for making number more meaningful and understandable.

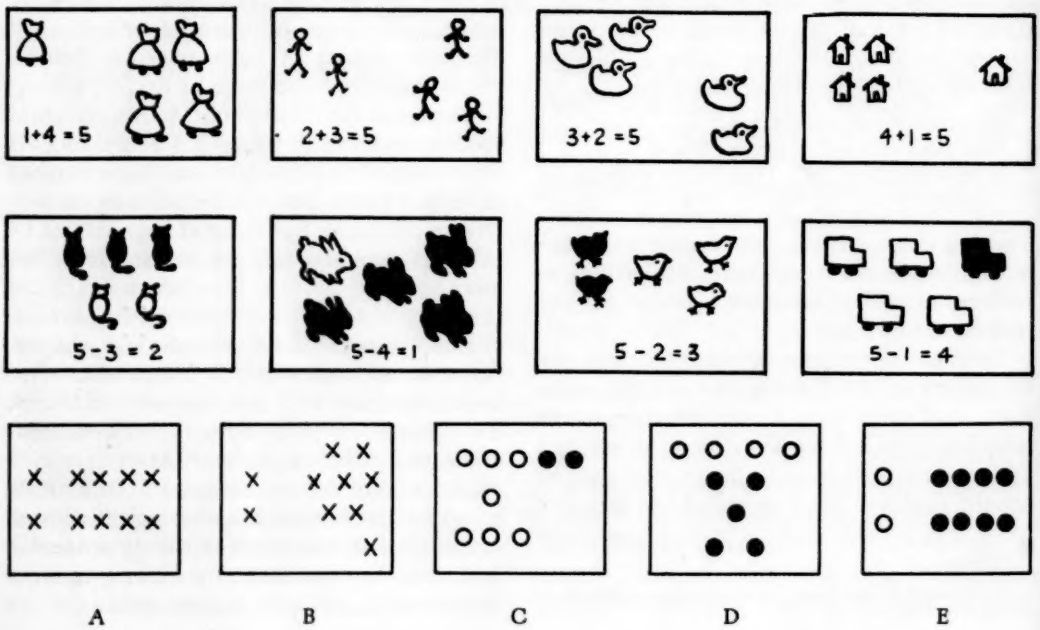
Small cut-out figures—ducks, elephants, cars, etc.—may serve in this connection. These are easily obtained, since they may be cut out by the children themselves, and if they are cut from heavy mounting paper, they will be sufficiently substantial for the desired use. These cut-out figures may be chosen to suit seasonal interests or to correlate with units of work. In any event, they provide many additional experiences in using the combinations, since they can be moved about just as are the concrete objects.

Semi-concrete number picture cards may be made by pasting small pictures in various designs to show the combinations. Advertising pages in current magazines in which the same little picture occurs repeatedly are well suited to the making of these cards. Some of the advertisements which make up into attractive picture cards are the little Dutch girls from the Old Dutch Cleanser ads, the cans of soup from the Campbell Soup ads, the boxes of jello from the Jello ads, and small cars from automobile ads. Of course, small pictures cut from patterns may also be used. Teachers sometimes have access to commercial printing presses, containing small pictures which may be stamped on cards. The printed cards may be made more decorative by the addition of water color or crayon.

In the first two rows below are some illustrative samples of picture cards for the number 5, showing the processes of both addition (top row) and subtraction (middle row).

For the larger combinations it is probably desirable to use only the regular group patterns as shown in the third row below. Otherwise, counting will be substituted for the addition or subtraction of groups. For example, the child can see Picture A as 2+8, because he has been taught to recognize the second group as 8, whereas it would be necessary for him to count part or all of the second group in Picture B.

Separate cards are suggested for addition and subtraction. The reason becomes clear when the pattern is considered. In addition the child combines two seen groups to secure an unseen total. To make the card mean 5−2 (or 5−3) the child must first know that the total is 5. If he gets the 5 through adding the 2 and the 3, he forms the habit of thinking of subtraction through addition. If he is told at the outset that there



are 5 objects, then his answer, whether 2 or 3, is visible. He has to *do* very little to secure the answer. The final solution of the difficulty is by no means clear. However, the proposals to be made, though tentative, at least have the recommendation of having proved valuable.

In the subtraction picture cards here shown the child is encouraged at first to see one group rather than two, as in addition. The number of objects to be taken away may be crossed out or they may be black as in the pictures. Now, in the case of Picture C, the child may be told that the total number is 9. Then he can see for himself that when he takes away the 2 black balls he will have left a regular group pattern of 7. Thus this card tells him "Nine take away two are seven." In Pictures D and E, the number to be taken away is shown in regular group form,[2] this being the larger number in each case.

Two points remain to be discussed—the first is, how these cards are to be used. They are to be used for flashcard practice on the combinations prior to drill on the abstract facts. The digits themselves not being represented, the child must perforce rely upon his understanding to secure his answers.

The second point is that eventually the children must come to see that ○○ ○○○ means, not merely 2+3, but 3+2, 5−2, and 5−3 as well. Two important gains are made —the children relate the four facts to each other, thus systematizing their knowledge, and they see that what answers they get depend upon what they themselves do with the part-groups.

Other Materials.—The child has no quicker or better way of understanding and representing the combinations to 10 than by the use of his own fingers, though this method has been rather generally banned for fear that if he once begins to count his fingers, he will forever after do so. Yet the child can be taught to see his fingers as groups in much the same way as he sees his sticks and acorn caps. There is certainly no harm, but rather much good, in the use of the fingers, provided proper precautions are taken. The child should be taught that these groups are but temporary aids, that he should progress as rapidly as possible to the stage where he will have no need for fingers or for any other similar device.

The value of pictures which may be used in connection with the teaching of the combinations should not be overlooked. The ads for P and G soap have been found especially good since they employ groups of children. One of these shows one child at the top of a slide and four children on the ground beside the slide. Two of the children are girls and three are boys. By using these facts, the combinations 4+1, 1+4, 5−1, 5−4, 2+3, 3+2, 5−2, 5−3 may all be shown by means of this one picture.

Drill.—At this point, if the preliminary stages have been sufficiently built up, the child is ready to memorize the combinations. This is accomplished by means of drill. It is unnecessary to discuss drill materials since every teacher is undoubtedly familiar with them. The small individual drill cards as well as larger cards for group instruction are both to be recommended.

Miscellaneous Suggestions

Diagnostic Testing Form.—Every teacher is familiar with the child who finds number almost too much for him, who regardless of carefully planned group instruction appears to be completely lost. This child cannot be helped until his basic difficulties are known.

[2] It is desirable that the child at this stage be able to recognize groups. However, if he is not able to do so, he should be allowed to find out by counting. His difficulty is evidence of his failure to understand the preceding stages and of his need for individual instruction.

One way to discover them is to have the child work "out loud," saying orally to the teacher everything that he usually says to himself in finding out the combinations. Of course, there must be no disposition to censor the methods reported, or the child will not truly report what he is thinking. For this individual testing the teacher should prepare a form with certain combinations listed at the left and with spaces for records opposite at the right. These spaces must be large enough to permit the teacher to record the exact words of the child. In this way she learns how he thinks of numbers—his stage of learning—and thus secures a basis on which to begin remedial instruction.

Number Picture Dictionary.—As the child learns about number, the building of a meaningful arithmetic vocabulary becomes important. A number dictionary of the type illustrated below may be made on a large sheet of tagboard and hung on the wall. It will be found helpful in the understanding of these arithmetical terms and also in the recognition of the words needed in reading arithmetical material.

first	second	third	fourth
1 one	2 two	3 three	4 four
+ add and	− subtract take away	∴ a group	—— a line × a cross

Number Outside the Arithmetic Lesson.—As has already been suggested, children should be encouraged to use number in describing their experiences. Arithmetic must not be merely a matter for a set period, to be thereafter a closed subject. On the contrary, children must become conscious of number and of its possibilities in their daily lives. Occasional overheard remarks illustrate that some children are growing into real consciousness of number. The children in one group often note the number of children or animals pictured in their reading books. One child made this remark during group reading: "Here are four children in the room already, and two more are coming in the door. That makes six children." At another time a little girl, who had a windmill cooky in her lunch, carefully placed her hand over 2 of the wings, saying as she did so, "Four take away two leaves two." On another occasion a little girl came to school with the very exciting news, "Miss White, my brother is ten years old today. I didn't get to give him but four licks this morning, but I'm going to give him the other six when I get back home."

In the last analysis, materials are a means to an end, and not an end in themselves. The ideas and suggestions presented in this article represent merely a limited number of materials and devices which have been found valuable in making number more meaningful and more vital to primary children. There are undoubtedly many other ways just as valuable and as practicable as those suggested here. These will occur to the ingenious teacher, provided that she understands how children learn number. Few needs on the part of children offer such a challenge to her energies and her wit.

Social Arithmetic in the Early Grades

MARION J. WESLEY
Elementary Supervisor, Watham, Massachusetts

IN recent years in the education of young children, there has been a strong trend toward the elimination of the abstract number content from courses of study in arithmetic in the early grades. Replacing this elimination, or in some cases supplementing its retention, there is an equally strong trend toward a greater use of number in so-called "natural situations" or "when the need arises."

Whether the development level of children in the primary grades favors the abstracting of a certain part of arithmetic from its social settings or not, the fact remains that the use of number in the child's home and community environment and the carrying out of his own plans and enterprises, is probably the most vital part of his arithmetic experience.

In many curricula, the most familiar examples of the active use of number in school are a few suggestive experiences in the fields of counting, saying rhymes, telling time, recognizing coins, and playing games.

Counting	Telling time
absent children	school time
milk money	meal time
bank money	bed time
materials for the group	
Games	Recognizing coins
ring toss	playing store
bean bag	identifying money
ball game	

Sometimes the lists are extended somewhat and labeled as "desired" activities because they touch the daily life of the child. Again, more pretentious experiences are suggested such as making a playhouse or post-office, or giving a party. All too often the teacher is not made aware of the varying purposes, values, and principles which underlie these number experiences; thus she is deprived of those basic understandings which give point and direction to intelligent leadership.

If we wish to capitalize the child's natural number contacts and provide or stimulate others, we must be aware of certain fundamental facts:

I. Growth in number knowledge and ability requires a background and constant setting in social experience.

The modern school leads the child through experiences in current social life, literature, and dramatic play as a prerequisite to learning to read. As this rich background of meanings brings a better understanding of the symbols in reading, so a rich and varied background in experiences involving quantity brings a better understanding of symbols in number. *Meaning* is of paramount importance in both fields, but as yet it has not been recognized as fully in the latter because number, unlike reading, reaches deeply into the abstract and is largely evaluated by computation.

The greater the variety of experiences in the life of the school, the greater the chance for variety and repetition of types of number experience. In short, opportunities for number depend first upon active life—doing something, going somewhere. The routine school soon exhausts its possibilities for number experiences. Children cannot carry on real enterprises without running headlong into quantitative relationships which they must settle or solve. Such learning experiences cannot be left to chance or accident.

II. Good guidance demands a knowledge of the various aspects of number and a realization of the value of cumulative experience as a working basis for the manipulation of numbers.

The teacher who depends upon a specified list of number activities to control the experiences of her class, may or may not provide for all the phases of number necessary to build up rich concepts of number—to apprehend numbers themselves. On the other hand, the teacher who depends upon class experiences to give number development al-

ways as a by-product, may fall short also, and the children will have only haphazard or vague number notions.

Planning both for satisfactory pupil participation in the daily life and affairs of the school and for proportionate experience in number demands an awareness of many factors among which are the following:

a. The child must be at times an active participant in number experiences; at others an interested onlooker; at still others, both.

b. The teacher must know the three aspects of number concepts described below. She must know too, the various phases of number and the numerous settings in which number appears if she is to recognize opportunities for number instruction and not do a hit-or-miss "incidental" job. These three aspects are:

(1). Functional idea of number or what it enables one to do—recording, communicating, acting under its control. Thus the child counts, reads and uses numbers for his numerous activities, buys and sells for money, uses quantitative measures, and keeps score.

(2). Materials to which number applies or the idea of units of various kinds. The child needs to build up clearer and clearer ideas of these in their vast range. "Six" as position or quantity with familiar objects such as dolls and pennies is not the same as six with reference to yards and pounds or to minutes and hours. A half differs not only in actual material but in quantity, size, weight, etc., according to what is divided. Even a placement like "up" or "back" varies with materials and purpose.

These concepts cannot all be developed in the primary grades but more effort can be devoted to their growth than is now common.

(3). The way numbers are known, or the mathematics of number knowledge. The child, beginning with counting, learns a number as a place in a series, as "how many" of something, as a basis for estimating magnitude, as the sum of parts, as so many times this or that, as a part of a certain whole, as a repeating unit of measure. Such knowledge is necessary for the child to begin and to increase his control of number; i.e., to use it for his own purposes.

The fact here emphasized is that the clearer the teacher's ideas of number and its uses, the clearer her recognition of number opportunities for children. In addition, the fact that a cumulative background on increasingly higher levels is necessary for understanding of certain processes and problems. "It is time enough to think of fractions when emphasis is put upon the study in the fifth grade," is not thinking in terms of the child's understanding.

III. Sensitivity to fine distinctions and shades of meaning in number concepts and vocabulary finds and creates many real opportunities for number experience.

Some Opportunities for Number Experience

Using number symbols for various purposes

Numbering the pages of a booklet or folded paper

Placing serial numbers on cards in a rack beside cards with directions, or phrases to be interpreted by reading and action; child chooses or answers by number

Reading specified questions on chart and giving answers arranged at top of chart by the number instead of the printed words

Giving a program to an audience and using numbered cards with the sequence of activities on the program

Placing each day the correct figure on the month's calendar sheet

Making a table of contents page for booklet, year book, or magazine

Using a table of contents

Making price tags and tickets

Recording date of some outstanding interest such as:

when seeds were planted

when bulbs were placed in a dark place

when the thermometer went to zero or below

when we went to the Children's Museum

Placing the date number on the month's diary sheet each day beside the item recorded

Recording the weekly growth of bulbous or seed plants

Making a number book with illustrations and written captions to show varying quantities such as:

5 cats 8 balls

half a cup of milk

a little tree and a big tree

a long train

a pair of mittens

Recording game scores on blackboard

Placing numbers on a clock face, post office boxes, houses in a play city

Writing number of votes on board for comparison

Using commercial ice card in dramatic play

Telling sizes of shoes, stockings, gloves, dresses, suits, caps

Recognizing and using money values

Making price tags for store, shop or market

Making tickets for trips, movies, dramatic play

Bringing money for milk—Weekly Readers

Using coins or play money to show the various combinations of coins which may be used for the preceding purchases

Listing staple articles that can be bought for 1¢, 2¢, 3¢, 5¢, 10¢

Listing standard charges or costs such as:

telephone—5¢	movie—10¢
weighing machine—1¢	parking charges—25¢,
bus fare—10¢	35¢, 50¢

FREE!

air for tires	museums
water for radiators	parks
library service	

Making and solving simple problems centering about social science knowledge—farm, garden, wool, shoes, transportation, bakery, toy or gift shop, 5 and 10¢ store

Telling about earning money, saving money, buying gifts, valentines, paying for admission tickets

Telling cost of various stamps, including special delivery, air mail

Making a stamp book, using cancelled stamps with money values indicated

Noting time in daily life

Discussing and learning the time and method of sounding the storm signal—no school

halfpast seven—	7:30 A.M.
halfpast twelve—	12:30 P.M.

2–2 sounded four times

Finding out certain time schedules in the community

stores open—	A.M.
stores close—	P.M.

Bus schedule—every 15 minutes, 30 minutes

Using three-minute glass (5-minute period) for washing hands, toilet, cleaning up, putting on wraps

Marking or indicating certain dates on calendar sheets: Children's birthdays in the month; holidays; date for trip, party, or assembly; date of planting seeds or bulbs.

Telling about time limits for parking automobiles.

Making a very simple time table—bus or train schedule, child's daily schedule

Reading time questions and responding by moving hands on clock face to correct position—What time do you get up? go to bed? come to school? does father come home?

Finding out and telling when the mail comes to school, to the house; when mail is collected from nearby box

Looking at the diary record and telling what happened yesterday, on the 15th, on the 21st.

Telling when one is late for school—morning or afternoon—using clock face to show

Watching clock at home for timing radio programs. Telling these times to children in school—showing on clock face

Reading and telling numbers according to social usage

Telling own house number with street. Showing the number on the blackboard.

Telling own telephone number. Showing on board how it looks in the telephone directory

Telling a route number of a main highway on a certain street

Telling automobile license number. Showing on board how it looks

Telling of numbers read on automobile trips:

- detour—500 ft. ahead
- measured mile—100 ft. ahead
- end of measured mile
- signboards—mileage to certain towns and cities

Finding pages in table of contents

Reading dials on a radio

Reading numbers on a dial telephone

Reading numbers on the thermometer

Looking at outdoor thermometer to read zero or below zero

Reading numbers on scales when being weighed

Reading prices from newspaper advertisements

Reading numbers in directions on blackboard for doing certain things

Calling friends on the telephone

Reading numbers in recipes

Telling mileage on father's car

Reading numbers on calendar

Using fractional parts

Following direction on board—2 (3) may draw here. Dividing space for two or more children

Talking about and estimating half fares for children

Telling sizes of shoes

Recognizing values of notes in music

Measuring quantities for cooking—part of cup, spoon, pound; painting—part of cup; construction—part of yard or foot, or merely "half of it, a quarter of it."

Telling ages

Playing store—using half or quarter of quantities

Giving fractional equivalents of a dollar for 25¢, 50¢

Folding paper into halves or quarters for special uses

Commenting on activity and usage such as:

We are halfway through the book.

I've used about three-quarters of my paste.

The line in the middle of the road warns cars to stay on the right side.

Measuring continuous quantities

Not measuring separate things but counting the number of times the unit of measure is applied to the thing measured

Using footrule or yardstick

Getting required distance for standing in throwing balls, bean bags; construction of costume, scenery; making loom for rug—half-inch spaces for brads; measuring own height; weighing oneself on standard scales.

Using measuring cup and spoon for making jelly, junket, cocoa, candy, cookies

Using units of measure

Bringing and using egg boxes to plant a dozen seeds; to hold a dozen Christmas tree ornaments

Bringing in pint and quart bottles for comparison with half-pint milk bottle in use

Watching indicator on gasoline pump register the number of gallons

See "Time"—"Money Values"

Using numbers as ordinals, cardinals, ratio

Reading from diary record. What happened on the 2nd? 10th? 20th?

Finding the designated page in a book: Turn to the fifth page.

Designating without touching as: I made the fourth picture. The third airplane is mine.

Running races. He came in second; fifth.

Telling dates of birthdays and holidays

Giving directions: It is the third street down.

Looking at pictures: What is the third monkey doing?

Checking spelling words in column: How many had the fifth word wrong?

Getting the required number of materials for a group of children

Sewing certain number groups of beads on an Indian costume

Giving the rhythmic beats on a tom-tom

Noting proportionate size of letters: The tall letters are twice as high as the small letters (or reverse)

Recognizing ratio of coin values

See—"Using Fractional Parts"

Counting for real needs

Plates, valentines, milk, books, for a specified number

Absent children

Making a notched stick or knotted string to record time or events (all the above may be one-to-one correspondence)

The number of days before a holiday or special occasion; votes; scores, using tally form

Checking the number of books in the library

Counting orally, one child after the other, to get exact number in the group

Some enterprises involving much use of number

Making and operating a post office, a store or market, a play house, a reading corner, a bus, train, boat, airplane, a play city, a wayside stand, a theatre, movie, puppet show, a garden—indoor and outdoor

Planning and carrying out a pet show, a circus, a party, a trip, a dramatization, an assembly program, the hatching of chicks, cooking and preserving, a Health Day program

Learning new games such as ten pins, Beano (modifications)

SUMMARY

1. If the child is to use his number knowledge intelligently, he must learn it intelligently—learn it with attention to *meanings*.
2. Arithmetic does not confine itself to figures alone. The school needs to pattern after life, where one scarcely ever works abstract computations; they are always part of a problem.
3. Social arithmetic is a vital part of the arithmetic curriculum. To dabble or touch lightly in this field may be better than to do nothing. But foundations of real understanding and use require systematic planning, effort, and checking of types of opportunity.
4. Wherever materials are handled there is quantitative measurement. At no time, however, should number be extracted at the expense of children's satisfactions along other lines.

Instructional Materials for Teachers

MARION PAINE STEVENS
Ethical Culture School, New York City

THESE instructional materials for teachers of arithmetic are classified under four headings: practical life materials under which are grouped various materials which involve number experience in life as well as in the activities of the classroom; materials for arithmetic only; games; and teachers' materials.

1. Practical Life Materials

Ruler—½-inch divisions in Grade I, ¼-inch divisions in Grades II and III

Yardstick, tapemeasure, trysquare—used in wood work. A five-foot measuring stick made in school is useful for measuring heights

Teaspoon, tablespoon, measuring cup—divided into thirds and quarters

Half-pint, pint, and quart milk bottles

Calendar—with large figures

Toy clock—small and large sizes—Real clock, stop watch, thermometer—the inexpensive thermometers obtained at the ten-cent stores are sufficiently accurate and may be handled more freely.

Toy money—to $1.00. Younger children should have only pennies, nickels, and dimes.

Used stamps for postoffice play

Scales and weights for store play

Dial telephone

Toy cash register

Sandglass—three-minute glass from ten-cent stores.

Compass—to be used in making circles

Paper cutter

2. Materials for Arithmetic Only

Abacus

One containing 100 beads in 10 rows of 10. The counting frame is as old as arithmetic and is still used in a large part of the world. It has many uses from the kindergarten on.

One-inch cubes, one-inch wooden squares

A plentiful supply for counting, or developing the combinations.

Sticks or splints, 4 to 6 inches long

Placed in bundles of 10 and 100, they have value in developing the larger numbers concretely.

Miscellaneous counters

Such as large dried beans, pumpkin seeds, or any similar small article which will not roll.

Domino cards. Hammett and others.

Sheets to be cut up which contain all combinations to 10. Set of 12 sheets, 20¢. Large domino cards with dots 1 inch in diameter, 50¢.

Washburne individual arithmetic cards. Chicago: Plymouth Press.

Boxed sets of 2- by 3-inch drill cards containing the 100 arithmetic combinations for each fundamental process. Inexpensive and durable. A corner is cut off each one, thus making sorting easy.

The Badanes Counting and Calculating Card. New York City: Continental Printing Company, 344 West 38th Street.

Two rows of ten dots each with sliding cover and pointer. Small card for pupils, 9¢. Large size for teacher, 52¢.

Thompson Self-verifying Matching Cards. Springfield, Massachusetts: Milton Bradley Company.

Set I. Combination 1 to 6. Set II, 1 to 10. Correct answers are matched to number combinations, an irregular piece being cut out of a square in such a way that only the right answer will fit.

3. Games

A distinction should be made between a game which is played for its own sake with number incidental and a drill or device which is for number practice. Games which involve considerable number are the following: (Most of them are best adapted to group play.)

Parchesi, lotto, fishpond, magnetic fishpond, jackstraws, magnetic jackstraws, dominoes, go-bang ("five in a row").

Flinch

A card game involving numbers. For Grade III and above.

Dice game

Two dice are thrown. The child giving the resulting sum first, adds one point to his score. If large dice are not obtainable, they may be made from tagboard.

Ring toss

Beanbag board

Cut holes of varying sizes in a board and lean it against the wall. Chalk the value (changing) of each hole below it. Beanbags may also be thrown into a box or waste-basket, or into circles chalked on the floor.

Number board or ring the hook

Screw cup hooks into a board and paint or paste a number beneath each one. Hang on wall. Children throw rubber preserving rings and add score.

Ten pins

Each pin knocked down may count one or a multiple of one. The usual game is of wood, but cardboard cylinders made of tagboard fastened with adhesive tape may be substituted.

4. Teachers' Materials

The Magic Number Press. Kansas City, Missouri: T. G. Nichols Company, City Bank Building, $10.95. Circular sent on request.

Rubber stamps containing words and groups of numbers in large clear type, which make over 1000 combinations. Prints a whole row or group of examples at once. Can be used on a duplicating machine. One of these in a school will save many hours for the teachers.

Printing Set

The numbers from a sign-marker printing set are useful for making materials.

Lettering Pens

At least two sizes for making individual and class material. The round-nibbed pen is advised.

A.C.E. Convention

Swampscott, Massachusetts

June 26-29, 1935

The Study Classes

A unique and outstanding feature of the Convention this year will be the eleven study classes meeting for two-hour periods on three consecutive mornings under the leadership of the following people: Ruth Andrus, State Department of Education, Albany, New York; Mary F. Babcock, Providence, Rhode Island; Helen Christianson, Manhattanville Day Nursery, New York City; Mary D. Davis, U. S. Office of Education, Washington, D.C.; Abigail Eliot, Ruggles Street Nursery School, Boston; Julia Hahn, Supervising Principal, Washington, D.C.; Grace Langdon, Specialist, Federal Emergency Nursery Schools, Washington, D.C.; Mae Peabody, State Department of Education, Albany, New York; and Laura Zirbes, Ohio State University, Columbus. Topics for discussion in the study classes were announced in the April issue of Childhood Education.

These classes will give Convention delegates opportunities for meeting leaders in the field of childhood education, for exchanging ideas through participation in class discussions, for help and guidance in individual problems, and for personal conferences with class leaders outside class hours.

Announcement from the Housing Committee

In addition to the rooms at New Ocean House, others in and near Swampscott and easily accessible to headquarters hotel will be available at the following hotels: Hotel Edison, Bank Square, Lynn; Hotel Hawthorne, Salem; The Bellevue, Beach Bluff, Swampscott; The Willey House Hotel, King's Beach, Swampscott. Arrangements for rooms in hotels should be made by correspondence with the hotel managers. A large list of rooms in private families will be on file at the desk of the Housing Committee at the New Ocean House. Assignments of such rooms will be made upon arrival of members who will be taken to their rooms by the transportation committee. Rooms in private families cannot be assigned by correspondence.

J. Asbury Pitman, *Chairman*
The Housing Committee

NEWS FROM HEADQUARTERS

MARY E. LEEPER

NEW A.C.E. BRANCHES

The number of new groups affiliating with the national Association for Childhood Education this year has now reached 29. The latest:

Evansville Association for Childhood Education. President, Blanche Jung
6 Washington Avenue, Evansville, Indiana

PROGRAM OF A.C.E. CONVENTION DISTRIBUTED

Preliminary programs for the 1935 convention of the Association for Childhood Education to be held at Swampscott, Massachusetts, June 26th–29th, have been mailed to contributing members and officers of Branches.

A strong program, the attractions of New England, and the increasing general interest in the education of young children point toward a registration of perhaps 2,000.

Write to A.C.E. Headquarters, 1201 Sixteenth Street, N. W., Washington, D. C. for the preliminary program and to the Headquarters of the convention, New Ocean House, Swampscott, Massachusetts, for your room reservation.

ANTHOLOGY OF VERSE FOR YOUNG CHILDREN

The Macmillan Company announces the publication of *Sung Under the Silver Umbrella*, a choice anthology of best verses, old and new, for younger children. This book has been prepared by the Literature Committee of the Association for Childhood Education of which Mary L. Morse is Chairman.

Those who have enjoyed the stories in *Told Under the Green Umbrella* and *Told Under the Blue Umbrella* will welcome this third volume of the series. Contributing members of the A.C.E. are entitled to a special discount when purchasing the first two books. The publishing code now in effect will not permit a special discount to A.C.E. members on the third and newest book. Those ordering copies for libraries may ask for the usual discount to teachers. Order from the Macmillan Company. The price of each volume is $2.00.

STATE SCHOOL FINANCE

The Research Division of the National Education Association announces the publication of brief descriptions of school finance systems in fifteen states. Statements for the remaining twenty-three states are now in course of preparation.

If you wish a summary for your own state showing the principle sources of revenue and the method followed in apportioning state school funds to local districts, write to the Research Division of the N.E.A., 1201 Sixteenth Street, N. W., Washington, D. C. Price 5¢ per page.

NEW YORK KINDERGARTEN ASSOCIATION REPORTS

The Research Committee of the Public School Kindergarten Association of the City of New York recently issued a sixty-four page pamphlet called *Notes on the Functioning of an Activity Curriculum in the Kindergartens and Kindergarten Extension Classes.* Here is a delightful, readable account of real experiences which kindergartners in New York City have had with children in different environments and at different age levels.

The study and experimentation leading to the publishing of this report began and developed under the leadership of Miss Luella Palmer. The published pamphlet is dedicated to her memory.

A SUMMER OPPORTUNITY

The meeting of the World Federation of Education Associations in Oxford, August 10th–17th, was announced in the March issue of CHILDHOOD EDUCATION. Previous to this conference Alida Shinn of Mills College, California, will conduct a tour of England, Scotland, Ireland and the Continent. This tour is being planned with special reference to those who wish to visit schools for young children in these countries. For full information address Miss Alida Shinn, Mills College P. O., California.

Curriculum Trends **by Dr. Laura Zirbes is now available. Price 35c.**
Order from A. C. E. Headquarters, 1201 Sixteenth Street, N. W., Washington, D. C.

BOOK REVIEWS

Editor, ALICE TEMPLE

EDITOR'S NOTE:—The following bibliography, a substitute for the usual book reviews, has been prepared by **Marion Paine Stevens,** Ethical Culture School, New York City.

1. BACKGROUND MATERIAL—HISTORY OF MATHEMATICS

The Story of Numbers, The Story of Weights and Measures. The Story of Our Calendar. Telling Time Throughout the Centuries. Achievements of Civilization Series, Numbers 2, 3, 4, 5. Chicago: American Council on Education, The Committee, 5835 Kimbark Avenue. 1932–'33, 10¢–20¢.

These four illustrated booklets enable a teacher to understand how arithmetic has developed from social needs. Through such information children may be introduced to its primitive stages, just as they trace the beginnings of the basic industries, in order to understand more clearly the life of today.

Two other books in this field which, though written for children, are of general interest, are:

Marshak, I. I. (M. Ilin). *What Time Is It?* Philadelphia: J. B. Lippincott Company, $1.50.

Mr. Marshak is the young Russian engineer who writes fascinating schoolbooks for Soviet children of the U.S.S.R. under the pen name of M. Ilin.

Smith, David Eugene. *Number Stories of Long Ago.* Boston: Ginn and Company, 1919, 60¢.

2. TEACHERS' BOOKS ON METHOD

Brownell, William A. *Development of Children's Number Ideas in the Primary Grades.* Supplementary Educational Monograph No. 35. The University of Chicago Press, 1928, $1.50.

Based on experiment and research from which the author concludes that children are hurried too abruptly from concrete to abstract work, and makes helpful suggestions as to the intermediate steps.

Buckingham, B. R. and Osburn, W. J. *Searchlight Arithmetic*—Introductory Book for Grades I and II. Boston: Ginn and Company, 1928, 36¢.

Clark, J. R., Otis, A. S. and Hatton, Caroline. *First Steps in Teaching Number.* Yonkers-on-Hudson, New York: World Book Company, 1929, $1.20.

Another book dealing entirely with the first two grades.

Morton, Robert L. *Teaching Arithmetic in the Primary Grades.* New York: Silver, Burdett and Company, 1927.

The only one of the general texts entirely on primary arithmetic.

29th Year Book of the National Society for the Study of Education—Report of the Society's Committee on Arithmetic. Edited by G. M. Whipple. Bloomington, Illinois: Public School Publishing Company, 1930, $3.00.

3. READING BOOKS IN THE FIELD OF NUMBER

The following books may be placed on the children's library shelf in Grade II for free reading and enjoyment.

Brown, J. C. and others. *Fun With Numbers.* Evanston, Illinois: Row, Peterson and Company, 51¢.

Hayes, I. M., Gibson, C. S., and Bodley, G. R. *Numberland,* Part I. Boston: D. C. Heath and Company, 1928, 64¢.

Peet, H. E. and Clapp, F. L. *Number Games and Stories.* Boston: Houghton, Mifflin Company, 1930, 76¢.

Smith, D. E., Luse, E. M., Morss, E. L. *Walks and Talks in Numberland.* Boston: Ginn and Company, 1929, 72¢.

Studebaker, J. W., Findley, W. C., Knight, J. B. and others. *Number Stories,* Book I, Book II. Chicago: Scott, Foresman Company, 1932–'33, 60¢–68¢.

Thiele, C. L., Sauble, I. and Oglesby, N. W. *My First Number Book.* Chicago: Rand McNally Company, 1927, 75¢.

Upton, C. B. *First Days with Numbers.* New York: American Book Company, 1933, 40¢.

4. TEXTS FOR CHILDREN

Badanes, Julia E. and Badanes, S. *A Child's*

Number Primer. New York: The Macmillan Company, 1929, 80¢.

Based on the principle that arithmetic should be taught by developing general ideas rather than a large number of particular skills. Counting, gaining of number concepts, and calculating developed through pictures (images) instead of through symbols. Opportunity for much pupil activity. Teacher's manual necessary.

Everly, L. L. *Oral Drill Book in Arithmetic—Addition and Subtraction Combinations—for Grades II and III*. Bloomington, Illinois: Public School Publishing Company, 1928, 25¢.

Thirty-two pages. Durable cardboard covers. A useful book for developing speed and accuracy after the combinations have been learned. Not a workbook.

NOTE: No attempt has been made to list the many excellent third-grade texts which are part of a coordinated series extending through the elementary school.

5. WORKBOOKS

The workbook is a new tool which has only recently come into general use for primary children. Workbooks save copying, provide for individual differences and individual study, save a teacher's time, and are scientifically built. Most of them cost little more than a pad of blank paper.

Summitt Experimental School. *The Alpha Individual Arithmetics*. Boston: Ginn and Company.

A pamphlet for each half year, beginning with first grade, which is both a workbook and an arithmetic text.

Smith, J. H. *Arithmetic Practice Exercises and Progress Tests*. Grades I and II. Chicago: Mentzer, Bush and Company, 1931, 24¢.

Of no value until all addition and subtraction have been developed. A few problems. Inside cover has a place for scores and graphs to mark progress.

Johnson, E. M., *My Progress Book in Arithmetic*. Columbus, Ohio: American Education Press, 1934.

No. 1 Easy addition and subtraction through 9.

No. 2 All addition and subtraction. Some multiplication and division.

No. 3 All four processes.

These books have many problems and interesting exercises, and a page for recording all scores.

Lennes Test and Practice Sheets in Arithmetic. New York: Laidlaw Brothers.

A workbook for each half year beginning with Grade II. Single sheets which can be torn out, having on one side examples and on the other problems and exercises. Small and rather crowded pages. Keyed so that children can make their own corrections and score their sheets.

Meyers, G. C. and Meyers, C. E. *My Workbook in Arithmetic*, Book I, Book II. Cleveland, Ohio: Harter Publishing Company, 1929–'30, 68¢.

For Grades I and II. Combine arithmetic and reading and give directions for pupil study.

Polkinghorne, Ada R. 1. *An Arithmetic Workbook for the First Half of Grade Two*. 2. *An Arithmetic Workbook for the Second Half of Grade Two*. 3. *An Arithmetic Workbook for the First Half of Grade Three*. 4. *An Arithmetic Workbook for the Second Half of Grade Three*. 5. *An Arithmetic Workbook for the First Half of Grade Four*. 6. *An Arithmetic Workbook for the Second Half of Grade Four*. Chicago: Ditto, Inc. Harrison Street at Oakley Boulevard, 1933. $1.50 per book.

The lessons in this series of books, sixty-four in each book, are printed in Ditto copying ink. By means of a Ditto duplicating machine one hundred copies of each lesson sheet may be easily and quickly reproduced by the teacher, thus necessitating the purchase of only one book for each half grade instead of as many books as there are children in the class. These lessons have been worked out in careful sequence. They are planned for children who have had much concrete experience with numbers as part of their previous work and play activities. They include both teaching and practice material and should be accompanied with numerous contacts with real situations involving the use of numbers.

Robertson, M. S. and Rugg, L. S. *Primary Arithmetic Pad*, Numbers One, Two, Three. Boston: D. C. Heath and Company, 1934, 24¢–32¢.

Large and well-spaced. Many problems.

Upton, C. B. *Arithmetic Workbook*, Grade I, Grade II. New York: American Book Company, 1933, 24¢.

Especially strong in developing number concepts. Large well-planned pages. Contains diagnostic tests and remedial exercises.

Washburne, C. *Individual Arithmetic*. Yonkers-on-Hudson, New York: World Book Company, 1934, 40¢.

A complete set of books for "self-instruction and self-correction" in which each child progresses individually at his own rate. Books I–IV are designed for the three primary grades. Teacher's Manual advisable.

AMONG THE MAGAZINES

Editor, ELLA RUTH BOYCE

EDITOR'S NOTE:—These reviews were written by **S. A. Martinsen,** Supervisor of Elementary Schools in Petersburg, Virginia, and **Miss Carolyn Cogbill,** Principal of the A. P. Hill School in Petersburg.

The following bibliography is not all inclusive but is intended merely to direct teachers to certain articles which the reviewers believe are helpful in the teaching of primary numbers. Three articles which are not in periodicals but are particularly pertinent to primary arithmetic at this time are also included.

"Project Arithmetic," Frank M. Rich. *American Childhood*, Volume 20: September, October, 1934 and January, 1935.

A series of three articles in which the author describes life-like activities through which he teaches the fundamentals in arithmetic:

1. "The Counting Habit" Vol. 20:13–14 S '34.

An explanation of how the author helps his pupils grow beyond the counting habit by objectifying numbers from 1 through 30 by means of natural experiences of the classroom such as the handling and arranging of materials, the playing of games, and the making of a health chart. He demonstrates the use of the abacus in teaching combinations without counting.

2. "Small Numbers and Combinations" Vol. 20:16–October '34.

Suggestions for building a toy store and cafeteria which the author believes furnish an abundance of practice with small numbers and combinations and help develop pleasant associations with numbers.

3. "Numeration to 100 and the Tables" Vol. 20:9–10 January '35.

Multiplication facts are developed through the use of an abacus and the pupils "have fun" with these facts through using them in a fish pond, a post office, and in games. The detailed description of the building and conduct of the post office show the many opportunities for both development and practice in these multiplication facts.

"Arithmetic." W. A. Brownell. *Childcraft* 5:121–155. Chicago: W. F. Quarrie and Company, 1935.

There are more failures in arithmetic and children generally have a more difficult time with it than with any other subject in the curriculum due to the type of instruction used in the lower grades. This results from an adherence by primary teachers to one of the two most prevalent theories: the drill theory or the theory of incidental learning. The author presents in this article, a third theory, namely, the meaning theory which insures progressive growth in quantitative thinking by means of systematic instruction in number in meaningful situations. The article contains a thorough discussion of the arithmetic program for the elementary school and gives many significant suggestions for translating the theory advanced here into classroom practice.

"What Is the Matter with Arithmetic?" F. M. McMurry. *Education* 54:449–51, April '34.

The author points out that while much improvement has been made in reading very little has been made in the field of arithmetic. He believes that the humanizing of arithmetic has suffered from the "strong desire of educators to be scientific"; this has caused the emphasis to be placed on the mechanics rather than on the thought or content of arithmetic. In conclusion he says that problems should be the center or "heart" of arithmetic and that the subject is progressing only in those schools where "tool phases are subordinated to purposes."

"Does Experience in the Situation Involved Affect the Solving of a Problem?" Helen M. White. *Education* 54:451–455 April '34.

The report of the results of a study based on 12 pairs of problems involving 24,000 solutions given to 1,000 children to determine the effect of experience or non-experience with a problem upon the success of the solution. The findings were that the amount of experience which the pupil may have had does not appear to be a large factor in the selection of the right process or the achievement of the right answer in easy problems but that lack of experience does appear to be a factor in the selection of the wrong process in easy problems; that the amount of experience is a very significant factor in the selection of the right process and the achievement of the right answer in more difficult problems, and that non-experi-

ence is also very significant in the selection of the wrong process, the omission of problems, and in the failure to complete the problems.

Therefore it is important that problems used in school, with the exception of the easy ones, be based on situations in which the child has had experience.

"Still More Ado about Zero." Foster E. Grossnickle. *Elementary School Journal* 33:358–364 January '33.

This author recognizes both uses of zero as given in preceding issues of this same journal: J. W. Dickey's "Much Ado about Zero," November 1931, and H. G. Wheat's "More Ado about Zero,"April 1932, first, that it is a number and, second, that it is a place-holder. In addition to this he believes that it has definite value in accurate measurement.

"An Experiment with a One-Figure Divisor in Short and Long Division." Foster E. Grossnickle. *Elementary School Journal* 34:496–506 March '34, Part I; 34:590–599 April '34, Part II.

Result of a study made with 2,365 pupils in Grades 5–15 to determine the superiority of the long- or short-division form when using a one-figure divisor. A test involving an element of time was given in two parts consisting of easy and difficult examples—Form A, short-division; Form B, long-division. The author recommends that only one form be taught—the long-division form—because his findings showed it to be superior. The short-division form may be used as a short cut after pupils have become proficient.

While this article reports a study of upper grade practice, it is significant for primary number in that, from the beginning, as pointed out the division facts themselves should be presented to prepare children for the long division form, thus

$$5)\overline{25} \text{ and not } 5)\overline{25}.$$
$$\quad 5$$

"Arithmetic and the Emotions." Garry C. Myers. *The Grade Teacher* 51:40+ December '33.

A discussion of the common practice of employing speed in the teaching of arithmetic showing the effect of its use on the emotions of the child. The emotional upset due to this "vice of speed" retards rather than aids learning; therefore, accuracy should be the main concern of the teacher in the beginning even though it requires more time. As the child gains in accuracy, speed will come.

"For 100 Percent Subtraction, What Method? A New Approach." Guy M. Wilson. *Journal of Educational Research* 27:503–508 March '34.

The results of the study of the use of twelve recognized methods of subtraction reported on 2,400 returned questionnaires from 23 departments of education, 162 cities, and 215 teachers colleges. The author recommends that the method most used—the take-away, borrowing upward method—be employed throughout the country since one method makes for a greater degree of accuracy which leads children to an attitude of confidence and success.

"Better Teaching of Arithmetic." Leo J. Brueckner. *The Journal of the National Education Association* 20: #7, 8, 9—October, November, December, 1931 and 21: #1, 2, 3, 4, 5, 6—January, February, March, April, May, June 1932.

A series of nine articles dealing with problems in the teaching of arithmetic. The author discusses at length: (1) the four functions of arithmetic, namely, the computational, the informational, the sociological, and the psychological; (2) the problem of general method in which he outlines a balanced instructional program in arithmetic; (3) the vitalizing of instruction illustrated by a description of a unit of work centering around preparations for a class party; (4) the nature of problem solving; (5) twelve major themes or generalizations about number which are basic to instruction in arithmetic; (6) the fundamental principles underlying the organization of practice work; (7) cause of failure in arithmetic and technic for diagnosing difficulties; (8) a remedial program; and (9) pupils' difficulties in problem solving and suggestions for improving ability.

"Remedial Cases in Arithmetic." William A. Brownell. *Peabody Journal of Education*, 7: # 2, September, 1929.

This article explains the four steps in the procedure employed in four case studies of children needing remedial instruction in arithmetic: namely, the measurement of initial status in arithmetic ability by the use of standard tests; the diagnosis of each child's weaknesses by studying the child's school records, by observing his reactions, and by noting specific difficulties as found from the Buswell-John Diagnosis Chart; provisions for program of remedial instruction and the determination of the effectiveness of instruction by means of final tests.

These case studies are significant to arithmetic teachers in that they demonstrate the value of a

teaching technique which deals directly with methods of thinking of number rather than with memorization of number facts which permits a child to adopt all kinds of inefficient procedures. Dr. Brownell points out that remedial instruction is necessary because of inadequate initial instruction.

"New Methods in Arithmetic." Margaretta Voorhees. *Progressive Education.* 5: 125–130, April-May-June, 1928.

Arithmetic is not a "patch" but is an integral part of the fabric of a child's life; therefore, the work in arithmetic in school should be approached in the manner, evidenced in his preschool life, in which the child learns it in life. In his first years he should learn number by using number in one of two ways: by actually using it in socially useful situations such as the school cafeteria or bank and through applying it to imaginary situations which may lead him beyond his actual experiences.

"One Reason Children Fail in Arithmetic." C. W. Washburne. *Progressive Education* 9:215–33 March, '32.

The report of the results of an investigation by the Committee of Seven to determine the cause of the large number of failures in arithmetic. The findings "showed that there was a definite time in a child's life before which it was inefficient to teach a given topic and after which a topic could be taught with reasonable effectiveness. This time could be determined by the child's previous knowledge of arithmetic and his mental age."

The committee discusses the various topics in arithmetic and suggests the time when each might be more effectively learned.

"The Psychology of Arithmetic." H. G. Wheat. *The Psychology of the Elementary School,* Chapter IV. New York: Silver Burdette and Company, 1931.

This chapter is divided into three parts: (I) The Development of Number, a genetic study of the number system, (II) The Child's Development in Number, and (III) Practice for Mastery.

Part II is a discussion of the way in which number ideas develop in a child's mind. It gives the four phases through which the first step in learning numbers, that of counting, is acquired. It shows the necessity for developing a thorough understanding of numbers before considering the relation between number ideas and explains the part that grouping plays in developing the "whole story" about a particular number and in building an arithmetical vocabulary concurrently. It presents the importance of the development of the ten idea and describes the method of employing it in developing the numbers from ten to nineteen particularly.

It contrasts the method of learning arithmetic by memorizing number facts with that advocated in this chapter. In the words of the author "the process is that of an active mind which not only learns the various number facts but also understands them and their relations and seeks possibilities for their application."

"Opportunities for the Use of Arithmetic in An Activity Program" Paul R. Hanna and others. Chapter V. *Tenth Yearbook of The National Council of Teachers of Mathematics.* New York: Bureau of Publications, Teachers College, Columbia University, 1934.

This committee begins its discussion of the subject by presenting three points of view held by advocates of the activity program:

(1) Arithmetic skill is developed adequately through functional experiences of children if those experiences are sufficiently rich and varied.

(2) There is a definite body of arithmetic content which must be taught and activities should be selected to motivate the teaching of these topics. Here the activity becomes a method rather than an end.

(3) Children should be taught arithmetic through activities which are meaningful and purposeful to them but there should be, in addition to this, direct instruction in arithmetic in a separate period for the purpose of fixing skills and providing for the logical development of aspects of arithmetic.

Since the committee reporting subscribes to the third of these points of view, it undertook to determine the soundness of its point of view by making a survey "to discover the extent to which opportunities for arithmetic in the activities program were possible." A list of all of the situations calling for the ability to handle numbers or to think quantitatively and a detailed analysis of the arithmetic problems in these situations were made by the teachers of the grades selected for this study. Groupings were made of the types of problems and of the skills which were involved in them.

The conclusion states that the activity program does not provide sufficient experience to enable a child to get the arithmetic that is necessary in the present social order but that the activities are useful in that they give meaning to number. A teacher is urged to study the needs of her own group and to provide many rich experiences for them.

RESEARCH ABSTRACTS

Editor, ELIZABETH MOORE MANWELL

Beito, E. A. and Brueckner, Leo J. "A Measurement of Transfer in the Learning of Number Combinations." *Twenty-ninth Yearbook of the National Society for the Study of Education.* Bloomington, Illinois: Public School Publishing Company, 1930, 569–587.

The subjects of this study were the children enrolled in three 2B grades. The experiment was continued over a period of four weeks. The main findings indicated that when children are taught as thoroughly as possible the direct forms of an addition combination, such as $\frac{7}{4}$, the reverse form, $\frac{4}{7}$, was learned at the same time.

Brownell, William A. "The Development of Children's Number Ideas in the Primary Grades." *Supplementary Educational Monographs,* No. 35. Chicago: The University of Chicago, 1928, Pp. xiv+242.

The investigation consists properly of four parts. A total of 1858 children from the low-first to the high-seventh grade were tested with respect to their ability to apprehend groups of objects numbering three to twelve as presented in objective patterns (Chapter II). The purpose of this first part was to discover the relative effect of the numbers in concrete form. The second part of the study deals with the manner in which children apprehend such number patterns (Chapter III). The varied methods they used are described, and the course of growth from clumsy, immature methods to efficient, expert methods is outlined. The third part (Chapter IV) is devoted to a statistical study of certain factors in the ability under investigation. The fourth, and by far the longest part (Chapters V–VII), consists of an analytical study of children's methods of dealing with the simple additive combinations and with learning to add three digits in a column. The actual methods employed in the course of their development are related to the corresponding treatments on concrete number. The monograph concludes with a chapter based upon the research findings and offers many suggestions for teaching number in the primary grades.

Brueckner, L. J. "Certain Arithmetic Abilities of Second-grade Pupils." *Elementary School Journal,* 27: 433–43, 1927.

The author tested in June and again in September some 200 second-grade children to see how much loss in arithmetic learning occurred during the summer. He found that there was very little loss in retaining the addition facts but considerable loss in the subtraction facts. There was less loss on the part of those pupils who had attended summer school. He found little differences in the test results of the boys and of the girls.

Buckingham, B. R. and MacLatchy, Josephine. "The Number Abilities of Children When They Enter Grade One." *Twenty-ninth Yearbook of the National Society for the Study of Education.* Bloomington, Illinois: Public School Publishing Company, 1930, 473–519.

This extensive study presents an inventory, by means of the teacher-interview technique, of 1356 children enrolled in schools in twelve different localities. Data are presented in twenty-eight tables. The results indicate that young children use numbers in their every-day life and have developed numerical concepts to an unexpected extent. One evidence of this is the fact that nearly half of the children in this study could find answers for half of the ten simple combinations which were tested.

Buswell, G. T. "Diagnostic Studies in Arithmetic." *Supplementary Educational Monographs,* No. 30. Chicago: The University of Chicago, 1926, Pp. x + 122.

A detailed study of 584 children of whom 114 were in the third grade. Three methods of analysis are employed. The first method, a complicated laboratory technique, consisted of a study of the eye-movements exhibited by children when adding columns of digits. The second method which can be used by properly trained teachers in the classroom consisted of recording the time consumed by individual pupils during the processes of addition, subtraction, multiplication and division and of comparing the records thus secured. The third method involved skillfully questioning and observing the pupil as he performed the four fundamental operations and thus securing a diagnosis of his mental processes.

To teachers perhaps the more valuable part of this study lies in the sections describing the sug-

gestions for remedial treatment of individual difficulties.

Court, Sophie Ravitch Altshiller. "Numbers, Time and Space in the First Five Years of a Child's Life." *Pedagogical Seminary*, 27: 71–89, 1920.

A parent's record of her son's interests and expressions concerning number, time and space during his preschool years. It is apparent from the record that his interest was spontaneous and all-absorbing, and that because of this interest and because of an obviously educationally-rich environment, he learned considerable factual knowledge. The author reports also that he always used most of his newly acquired knowledge in his work and his play as soon as it was learned and that he seldom forgot any of it.

It was found that there were seasonal variations in his interest and his learning, little interest in numbers being shown in spring, but special interests in concepts of time and space. For example, his ability to tell time was stimulated by his observation in the spring of the passing of trains and their schedule. The implications of this study for education are that, wisely guided, the young child may learn an astonishing amount and that formal pressure may be far less effective than spontaneous interest in the learning.

Court, Sophie Ravitch Altshiller. "Self-taught Arithmetic From the Age of Five to the Age of Eight." *Pedagogical Seminary*, 30: 51–68, 1923.

The author continued her study of her young son in his school and his out-of-school arithmetic experiences. She found a vivid interest in learning when the interest was self-imposed, but little carry-over into home activities of the arithmetic disciplines of the school where the motivation was imposed from without.

Judd, Charles H. "Psychological Analysis of the Fundamentals of Arithmetic." *Supplementary Educational Monographs*, No. 32. Chicago: The University of Chicago, 1927, Pp. x + 122.

Chapter II contains a detailed analysis of counting by adults. Numbers as high as 20 were presented by means of sounds and of flashes of light at varying rates. The complexity of the act of counting is convincingly demonstrated, thus serving to present new light on a skill which by common consent has been regarded as exceedingly simple. The measures represented were obtained through the use of elaborate laboratory equipment. Chapter III is similar. In this chapter data are reported for 120 children selected from grades I to VI in such a way that half represented the best pupils in their groups and half the poorest pupils. The complexity of the act of counting and its consequent demands upon children are again demonstrated, this time by the addition of valuable notes on the bahavior of the children who counted. Chapter IV piles up more evidence on the same points through the analysis of the individual cases. In Chapter V large numbers, number combinations, and problem-solving are discussed in terms of their historical development and in terms of their treatment in current textbooks. The new difficulties introduced by textbook problems are thoroughly described. The concluding chapter is entitled "Outlines of a Psychology of the Fundamentals of Arithmetic."

Knight, Frederick B. and Behrens, Minnie S. *The Learning of the 100 Addition Combinations and the 100 Subtraction Combinations*. New York: Longmans, Green and Company, 1928, Pp. 82.

An investigation which reports the number of practices together with time of responses for a group of twenty-five second-grade children in learning the elementary facts of addition and subtraction. The difficulty ranking of each individual fact is presented.

Neulen, Leon Nelson. "Problem Solving in Arithmetic"—A Study of Certain Factors in the Allocation of Arithmetic Problems Involving One, Two, Three and Four Steps or Processes in Reasoning. *Columbia University Teachers' College Contributions to Education*, No. 483. New York: Teachers College Bureau of Publications, 1931, Pp. 87.

The purpose of this study was to discover what constitutes current practice in the grade placement of problems with respect to the number of steps involved, to determine the ability of beginning pupils in grades three to seven to get correct answers to written problems involving one, two, and three and four steps, and to determine the relationships between such factors in problem solving as reading ability, computational skill and intelligence. More than 2000 children were tested, including 274 from the third grade. Among other more technical conclusions the author states: "The tendency in certain schools, especially in those where considerable attention is given to projects and activities, to decrease the amount of time given to computation, seems destined to lead to disappointing results."

Norem, Grant B. and Knight, F. B. "The Learning of the 100 Multiplication Combinations." *Twenty-ninth Yearbook of the National Society for the Study of Education*. Bloomington,

Illinois: Public School Publishing Company, 1930, 551–568.

The aim of this investigation was to determine the amounts of practice needed by typical third-grade children to learn each of the 100 basic multiplication combinations and to maintain this learning. From this date the relative difficulty of the combinations was determined. Twenty-five third-grade children were the subjects.

The main findings indicated that when mastery has been well established, one practice a week is enough for maintenance. Also it was found that the size of the product of a combination was a partial index to its difficulty. About 91% of the errors were the correct answers to closely allied combinations.

Smith, Nila B. "An Investigation of the Uses of Arithmetic in the Out-of-School Life of First-Grade Children." *Elementary School Journal*, 24: 621–626, 1924.

A study of 500 children to find out how much and in what form they used arithmetic in their out-of-school life. It was found that 30% of the occasions for using arithmetic occurred in transactions carried on in stores, 18% in games involving counting, 14% in reading Roman numerals on the clock, 13% in reading Arabic numerals in finding pages in a book and the remainder in many other ways. Of the processes thus involved 35% were in addition, 23% in counting, 12% in subtraction and 8% in fractions.

Taylor, Joseph S. "Omitting Arithmetic in the First Year." *Educational Administration and Supervision*, 2: 87–93, 1916.

A report on two years' experimentation omitting the formal teaching of arithmetic in the first grade in a large city public school. "The classes which omitted number work during the first year of school not only held their own against those that had numbers, but actually outstripped their competitors. Hence it is hard to escape the conclusion that the time now given to arithmetic in the first year is worse than wasted, for when that time was devoted to English in P.S. 16, the children were able to read about three times as much matter as classes of the same grade where arithmetic was studied. At the same time there was no loss of arithmetical ability since the children were able easily to make up during the second year the little arithmetic they had missed during the first year." (See the Buckingham article in this issue of CHILDHOOD EDUCATION for a different interpretation of the results.)

Wilson, G. M. "Motivation of Arithmetic." *Bureau of Education Bulletin*, No. 43. Washington, D.C.: Government Printing Office, 1926, Pp. iv+60.

The author collected, and presents in this study, from 5000 teachers a list of games and devices which they found useful in providing number experiences for their children in situations as nearly real as possible. Saving banks, grocery stores and many other practical matters are suggested.

In the June Issue

Owing to lack of space two articles planned originally by Dr. Brownell to appear in this issue will be published in June: "Arithmetic Meanings," by Amy J. DeMay, Mathematics Instructor in the Children's Village at Dobbs Ferry, New York, and "The Value and Limitations of Drill," by Lorena B. Stretch, School of Education, Baylor University, Waco, Texas. Other contributors will be:

William H. Kilpatrick, "Inclusiveness and Continuity in Educational Progress."

John Hockett, "The Primary School of Tomorrow."

Garry Cleveland Myers, "Where Shall I Live?"

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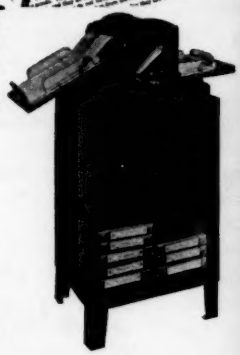

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